Formal Methods with Uncertainty

and other things.

Institute of Science and Technology Austria

Stochastic Systems

Stochastic Systems

Statistical Monitoring

- *Markov Chains (CAV 2023)*
- *Hidden MC (RV 2023)*
- *Linear Systems (FAccT 2023)*

Stochastic Systems

Planning

- *Abstraction-based Planning (CAV 2024 - under submission)*

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- *Planning under Statistical Uncertainty (ongoing)*

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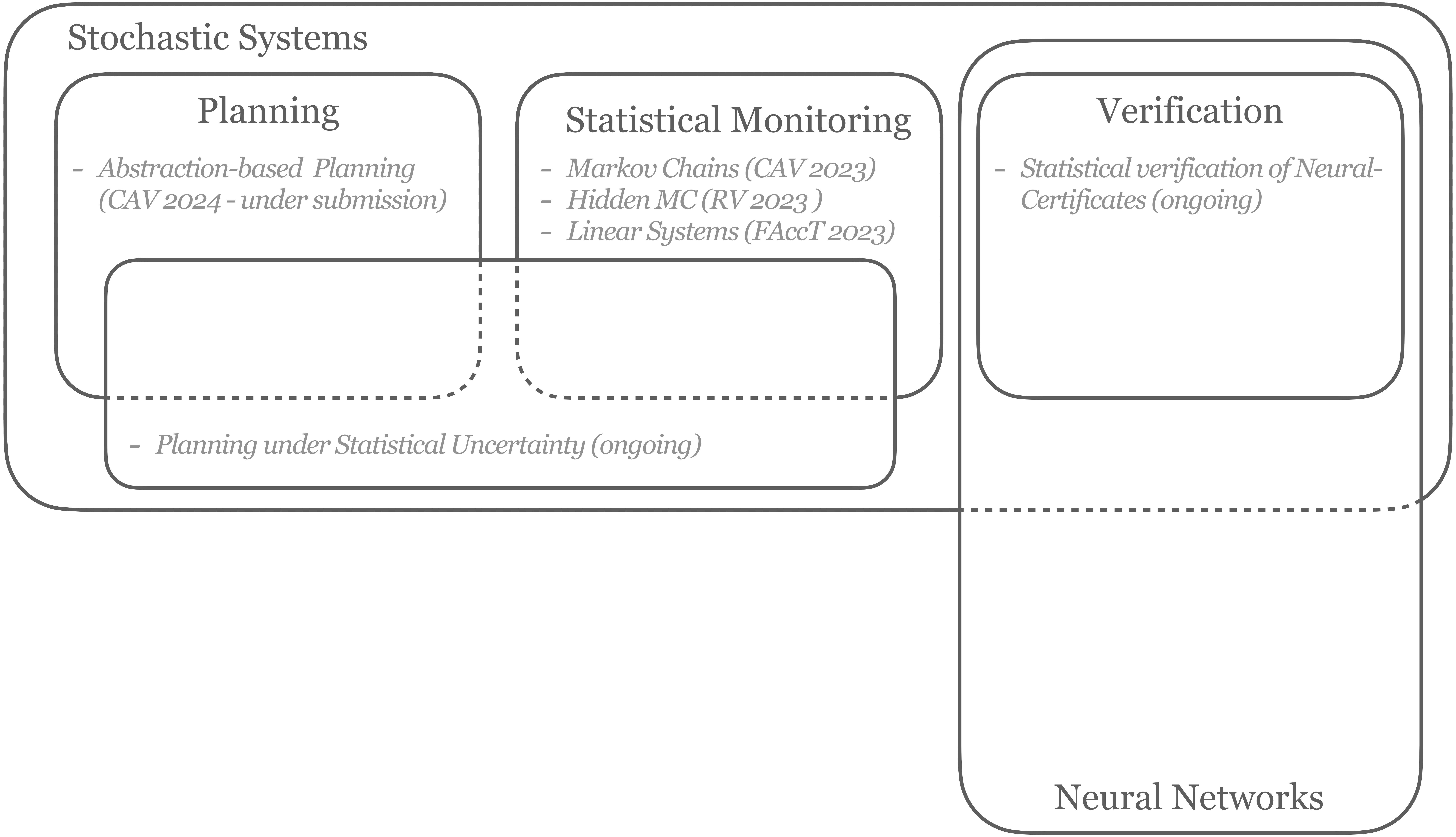
Verification

- *Statistical verification of Neural-Certificates (ongoing)*

- *Planning under Statistical Uncertainty (ongoing)*


Stochastic Systems
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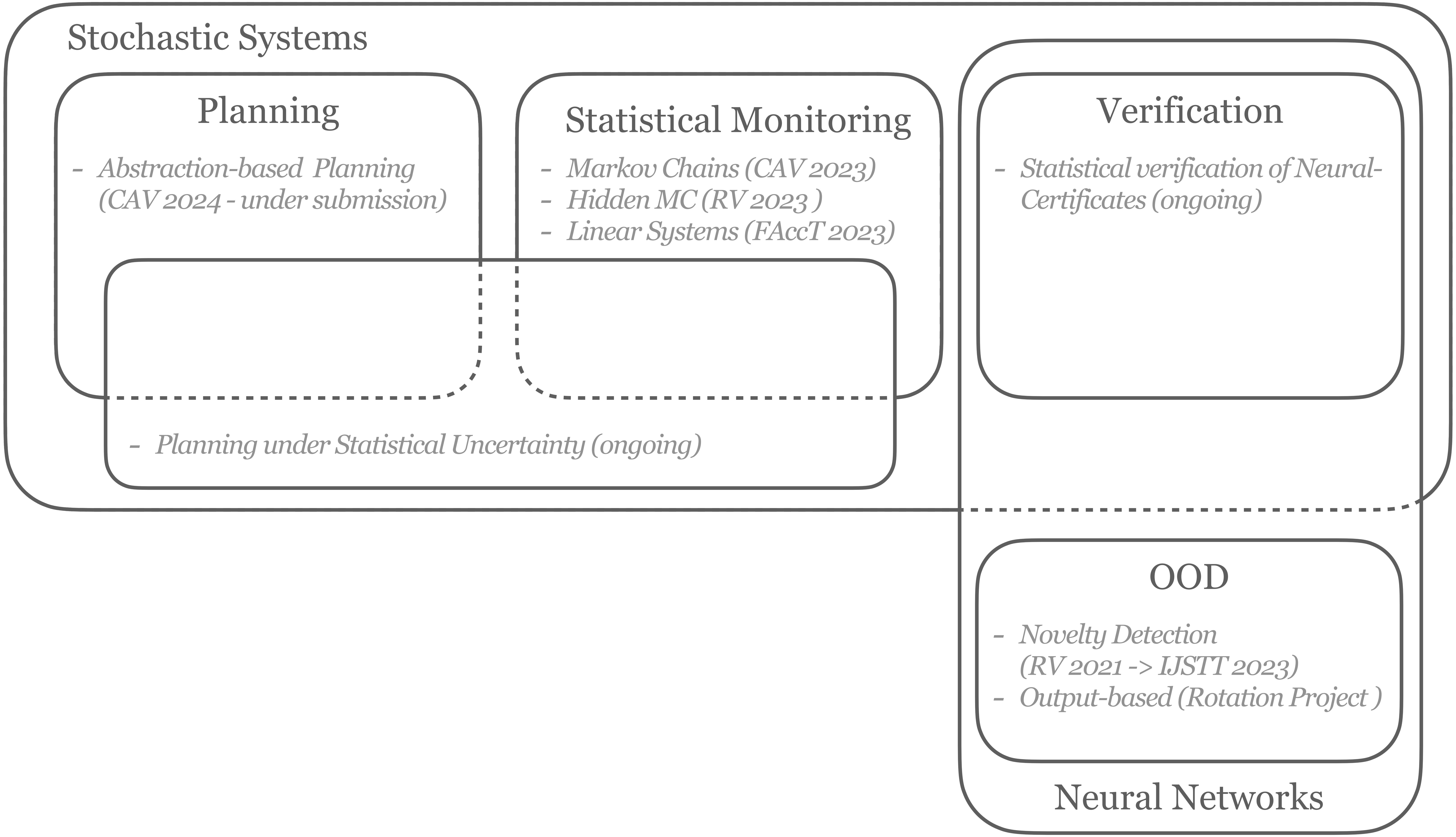
Neural Networks


Stochastic Systems
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- Planning under Statistical Uncertainty (ongoing)
Verification
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OOD
- Novelty Detection (RV 2021 -> IJSTT 2023)
- Output-based (Rotation Project)
Neural Networks

Quantify Uncertainty…

…for online monitoring;
…for planning;
…for verification.

Possible Directions.

At first glance.

Use statistical monitoring to quantify and reduce the uncertainty in the world model.

Improve safety in planning by quantifying state and/or model uncertainty.

Investigate out-of-distribution detection (OOD) in a sequential setting.

Statistical Monitoring…

…of Stochastic Systems.

$$\overrightarrow{X} := \left(X_t\right)_{t>0}$$

a stochastic process

$$f : \Sigma^* \to \mathbb{R}$$

some function

$$t \in \mathbb{N}^+$$

at any point in time

$$\vec{x}_t := x_1, \ldots, x_t$$

observe a realisation

$I \subseteq [1; t]$

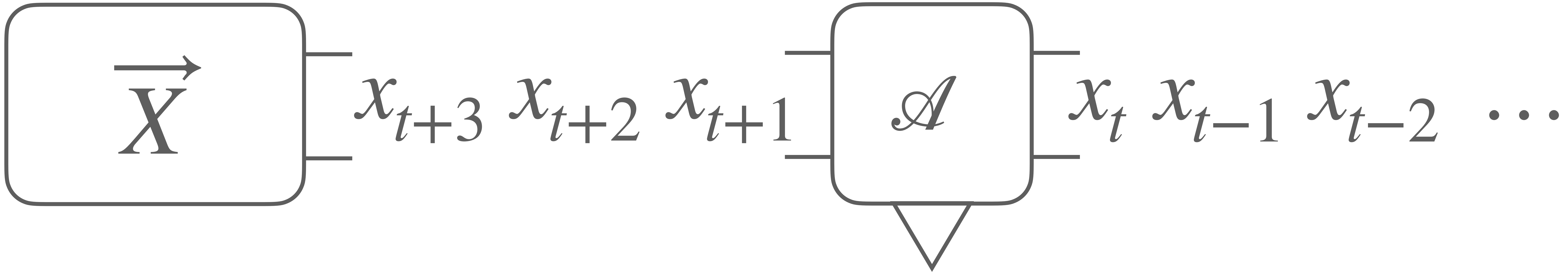
$$\mathbb{E}(f(\overrightarrow{X}_t) \mid \vec{x}_I)$$

want to compute

Limited information.

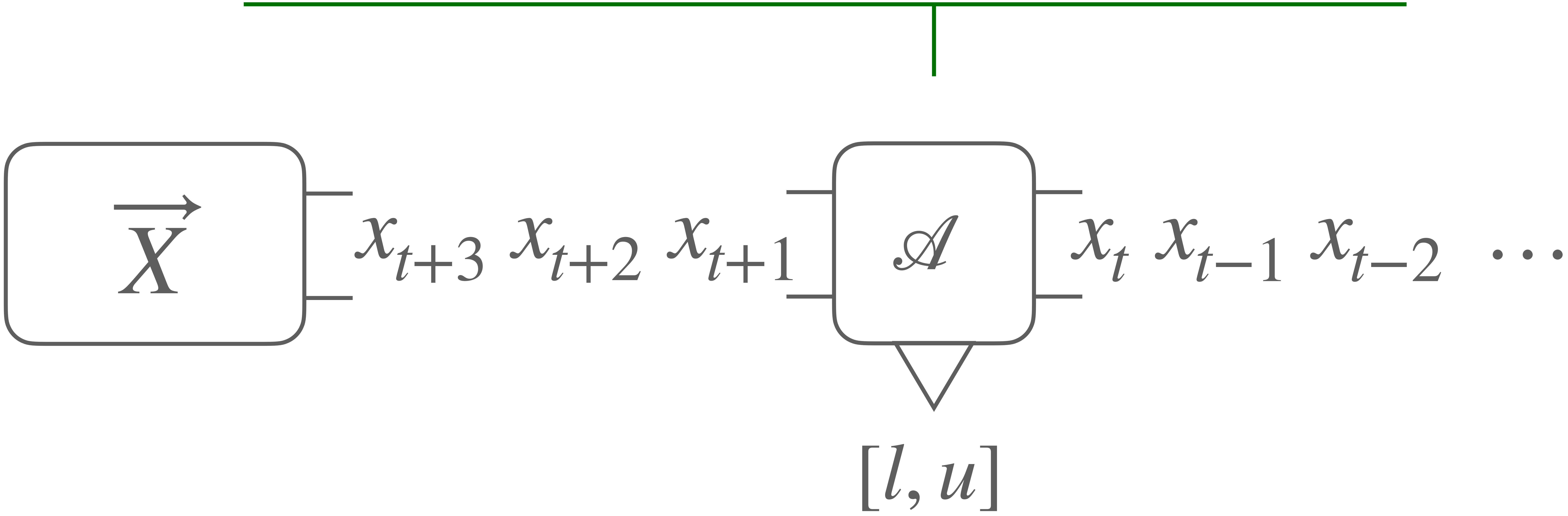
Estimate and quantify Uncertainty.

$$\overrightarrow{X} \quad x_{t+3}\ x_{t+2}\ x_{t+1}\ x_t\ x_{t-1}\ x_{t-2}\ \cdots$$



$\overrightarrow{X}$
x_{t+3} x_{t+2} x_{t+1}
$\mathscr{A}$
x_t x_{t-1} x_{t-2} $\cdots$

$$\mathbb{E}(f(\overrightarrow{X}) \mid \vec{x}_I) \in \mathscr{A}(\overrightarrow{x_t}) \text{ with probability } 1 - \delta$$



Example.

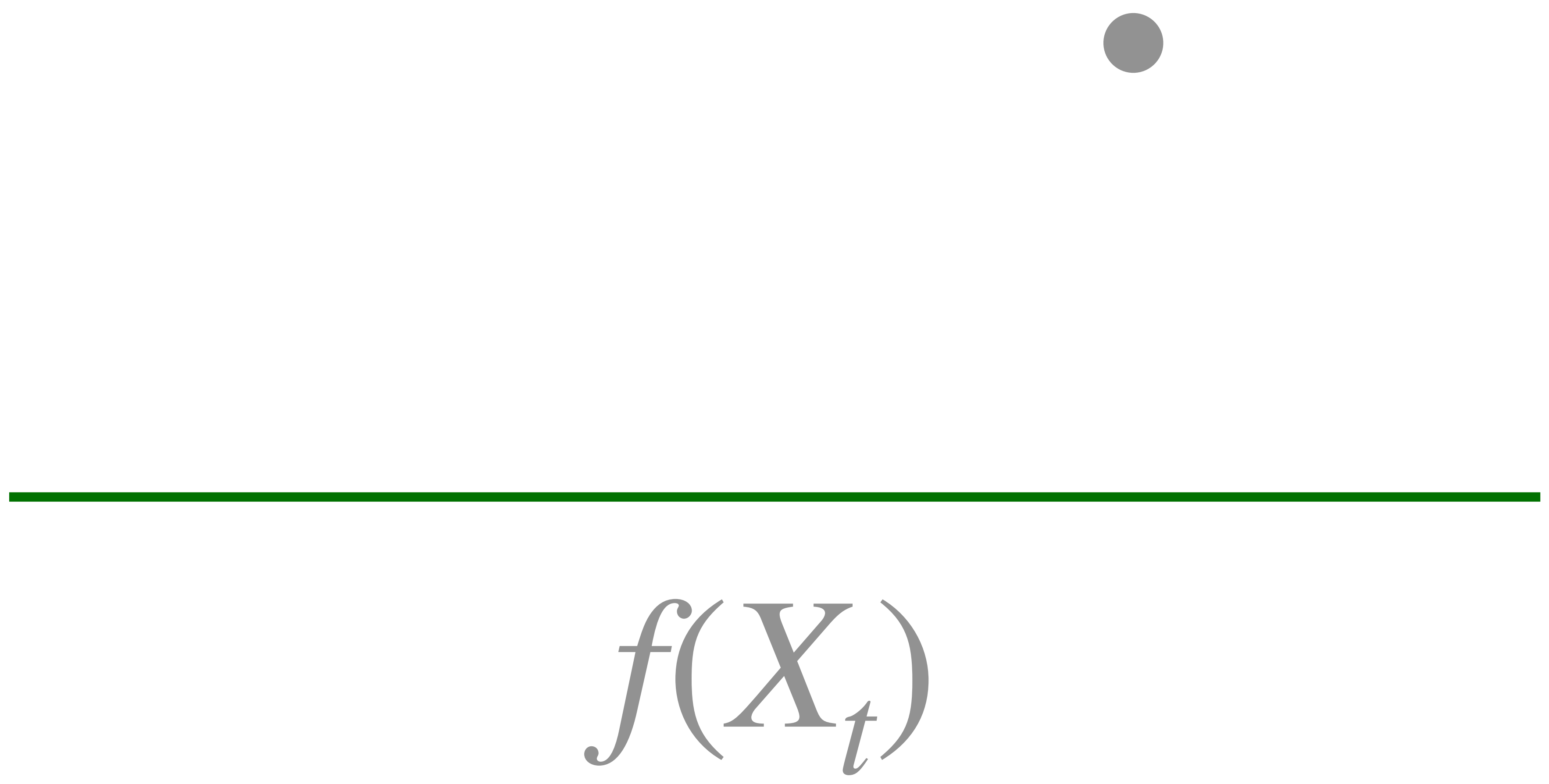
Bound uncertainty in parts of the world model, e.g. position of AV.

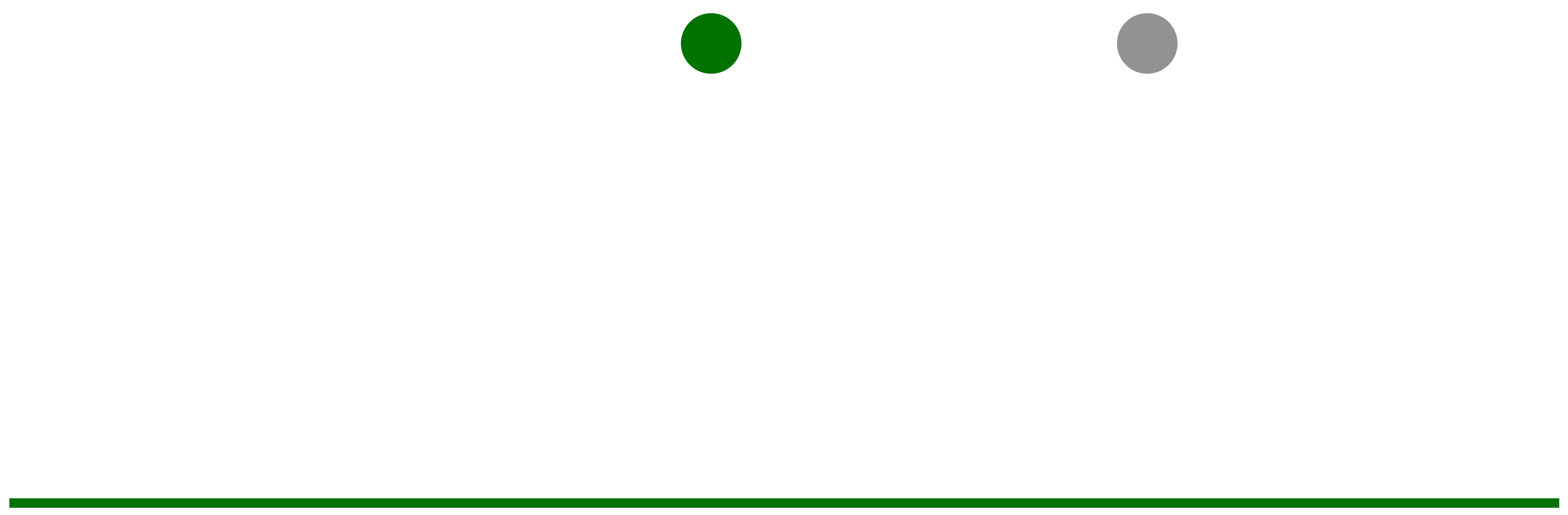
$\overrightarrow{X}$ … sequence of sensor measurements

f … sensor measurements to position

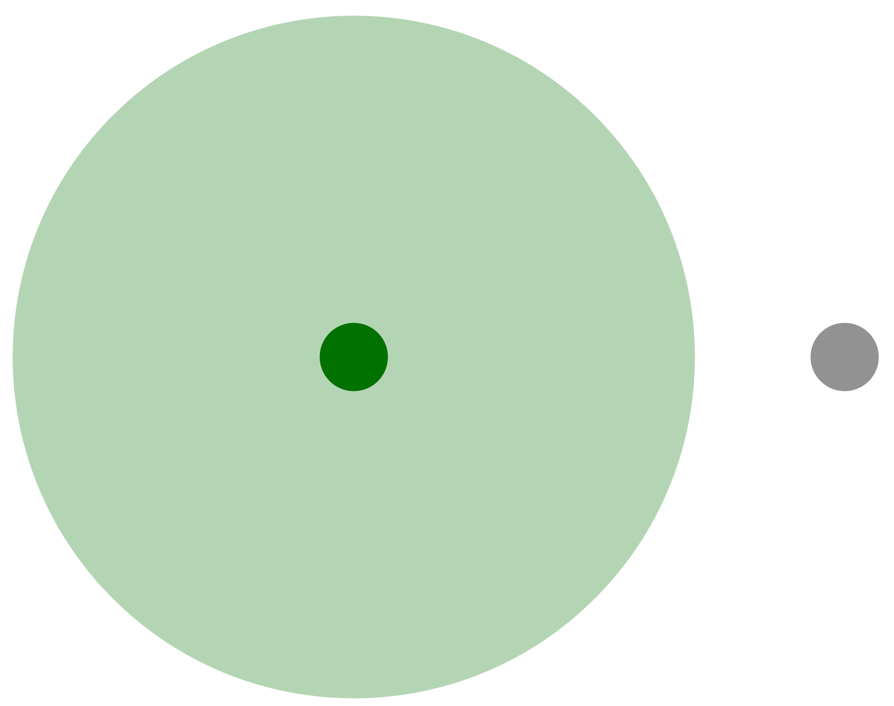
$$\mathbb{E}(f(X_t) \mid \vec{x}_{t-1})$$

Actual position at time t

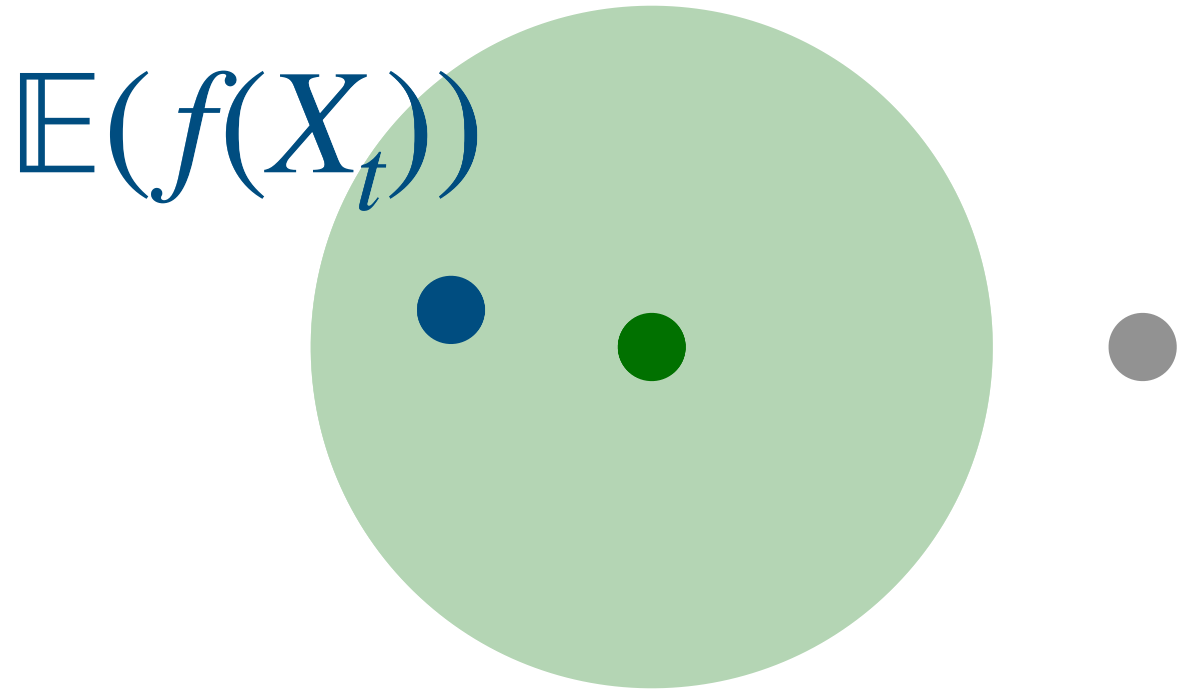

$f(X_t)$



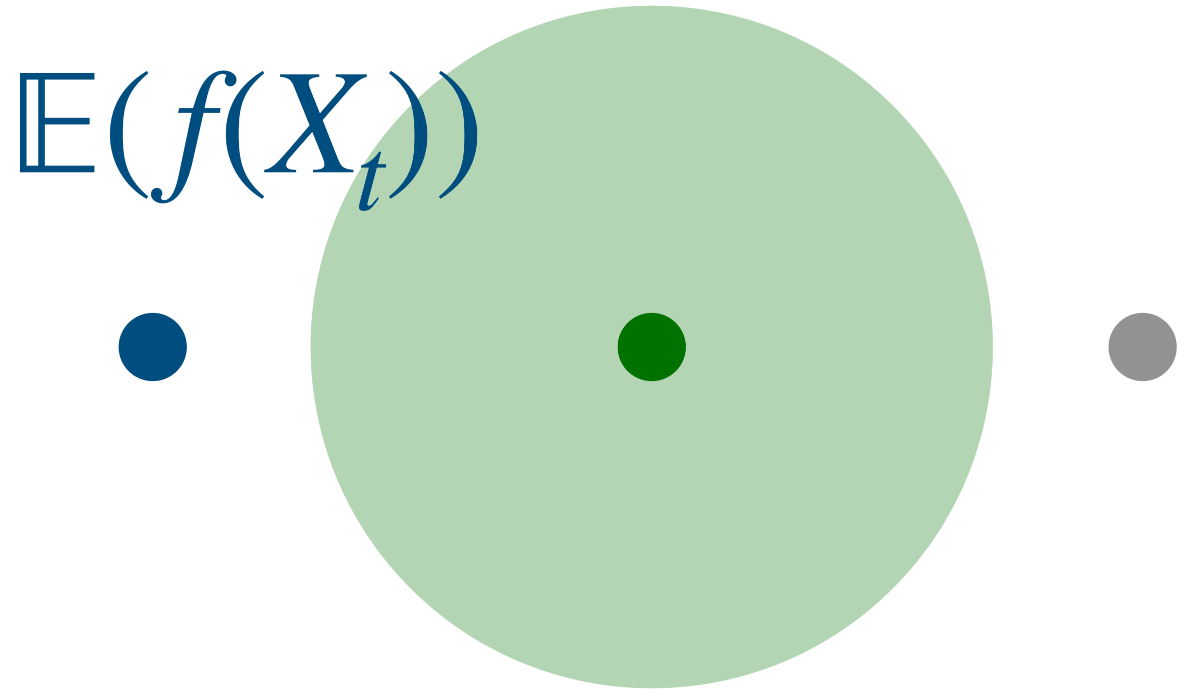
$$\hat{f}(\overrightarrow{X}_t)$$



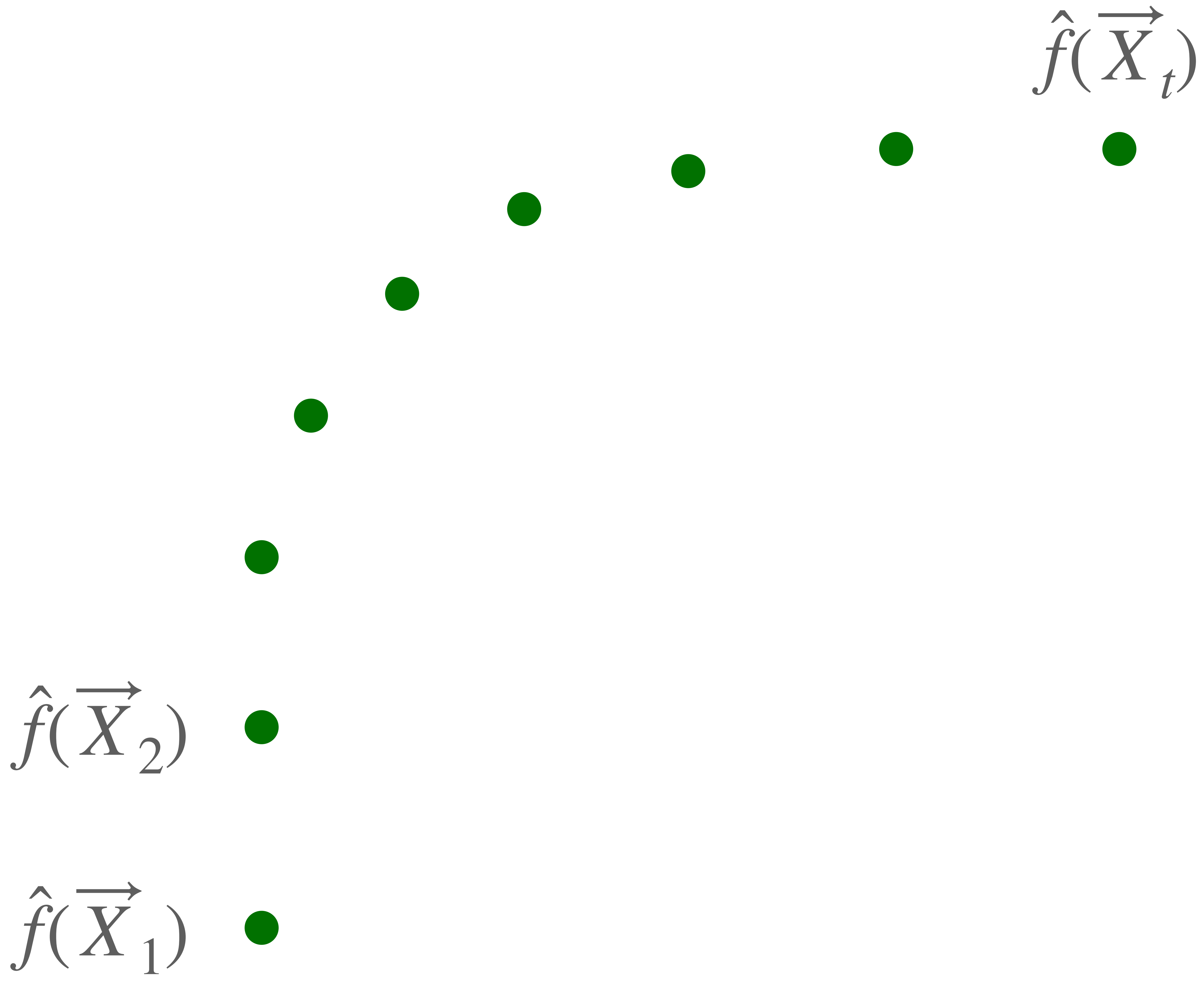
Find an ε such that …

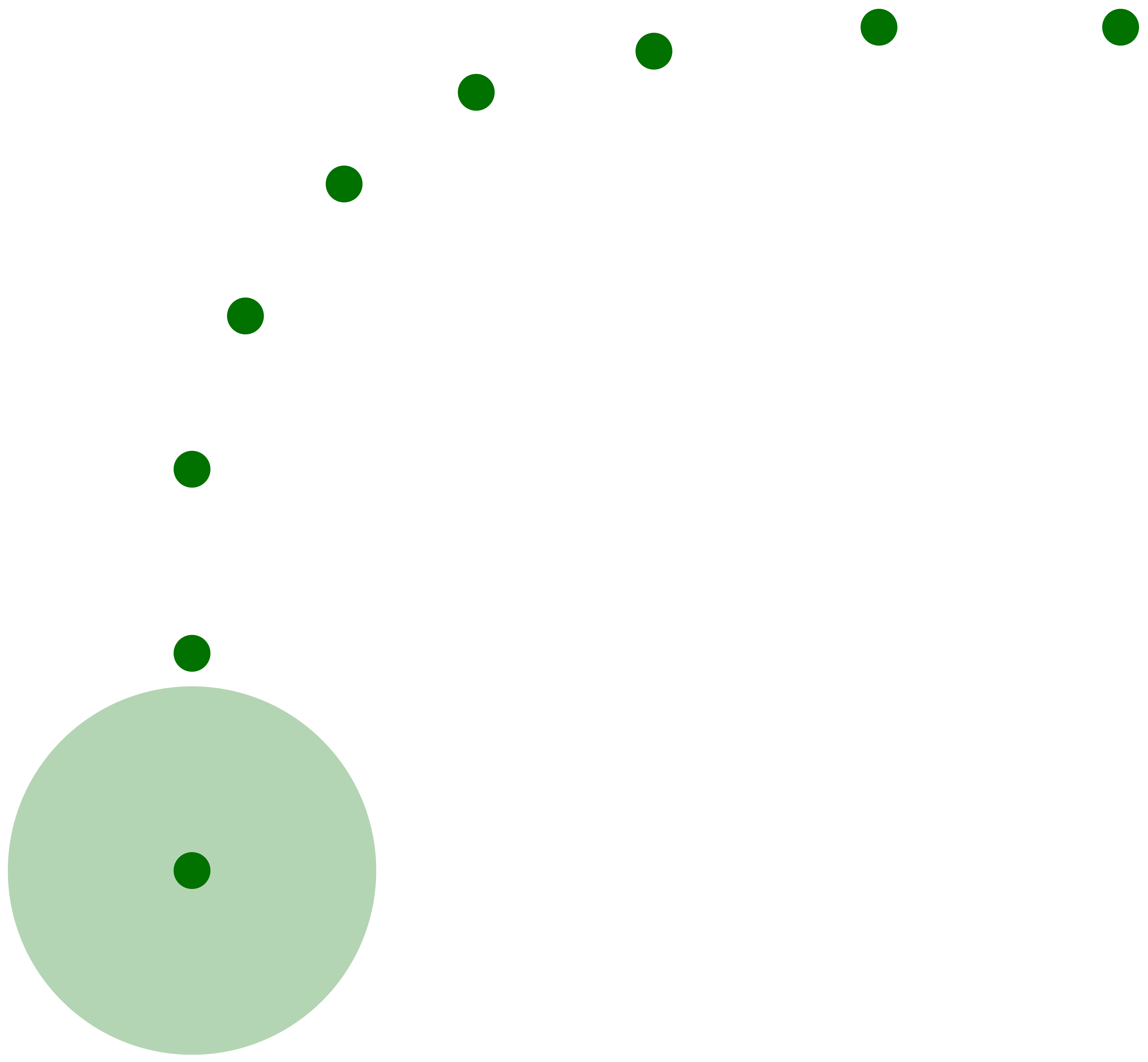


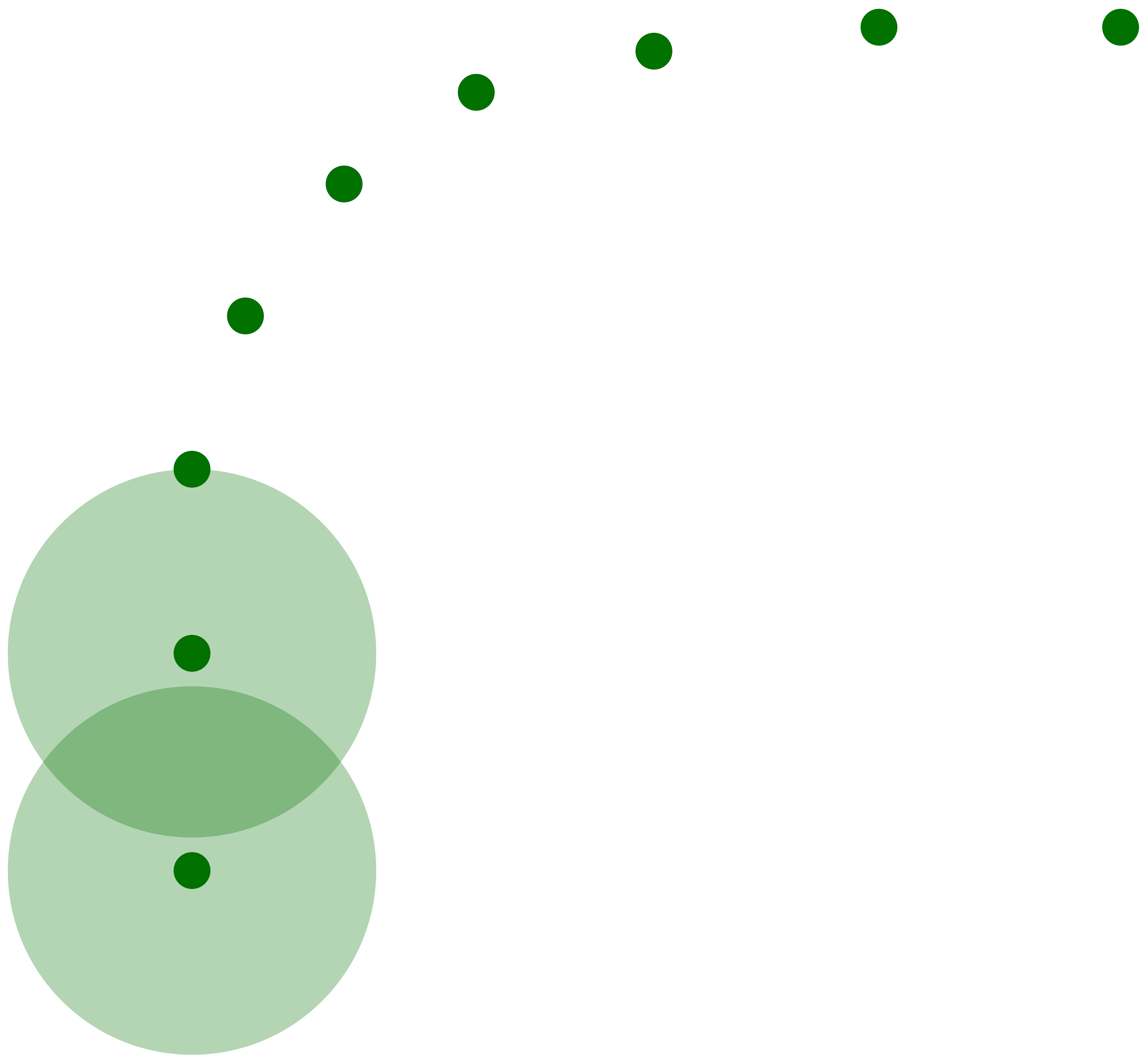
With probability $(1 - \delta)$

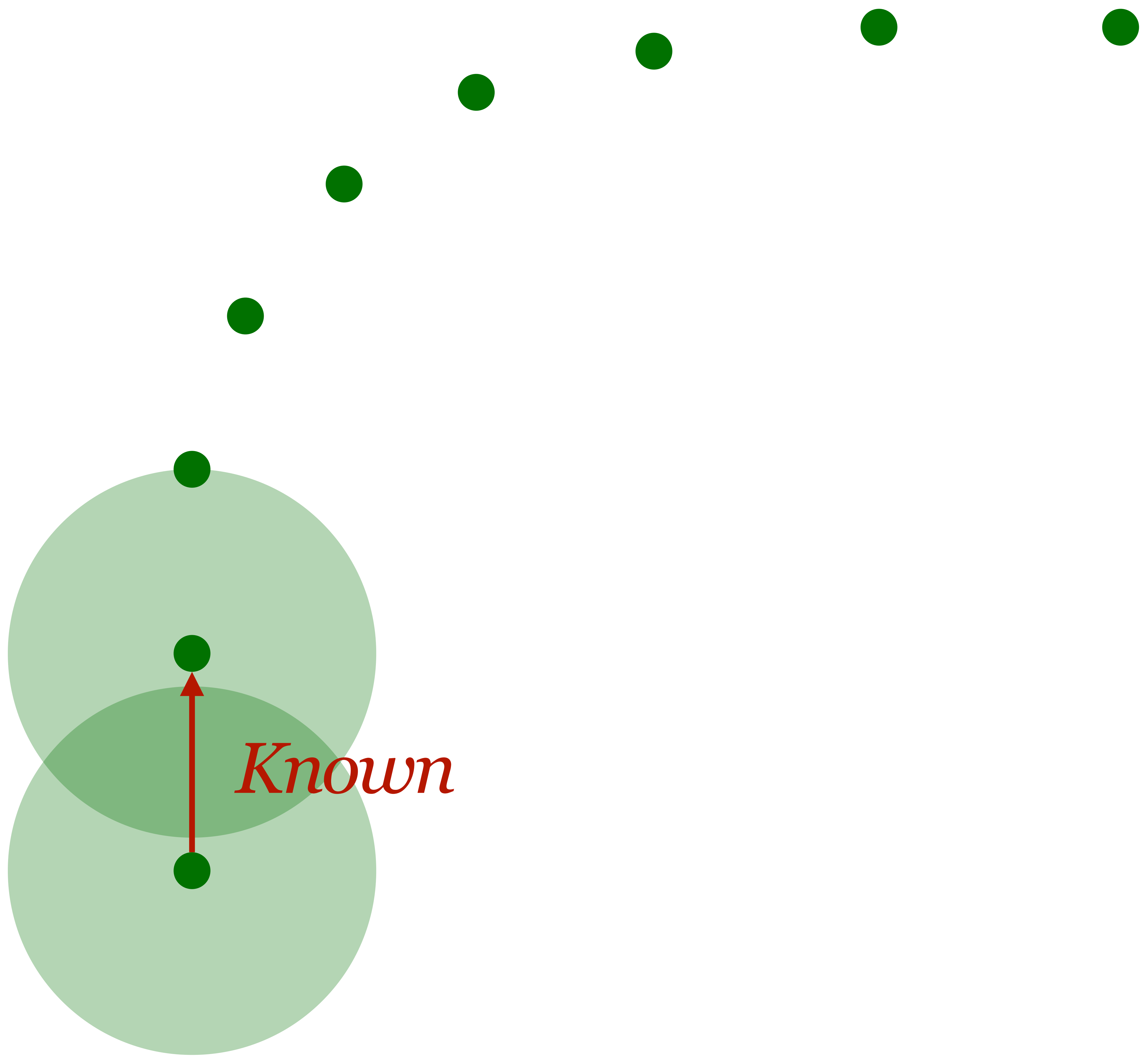


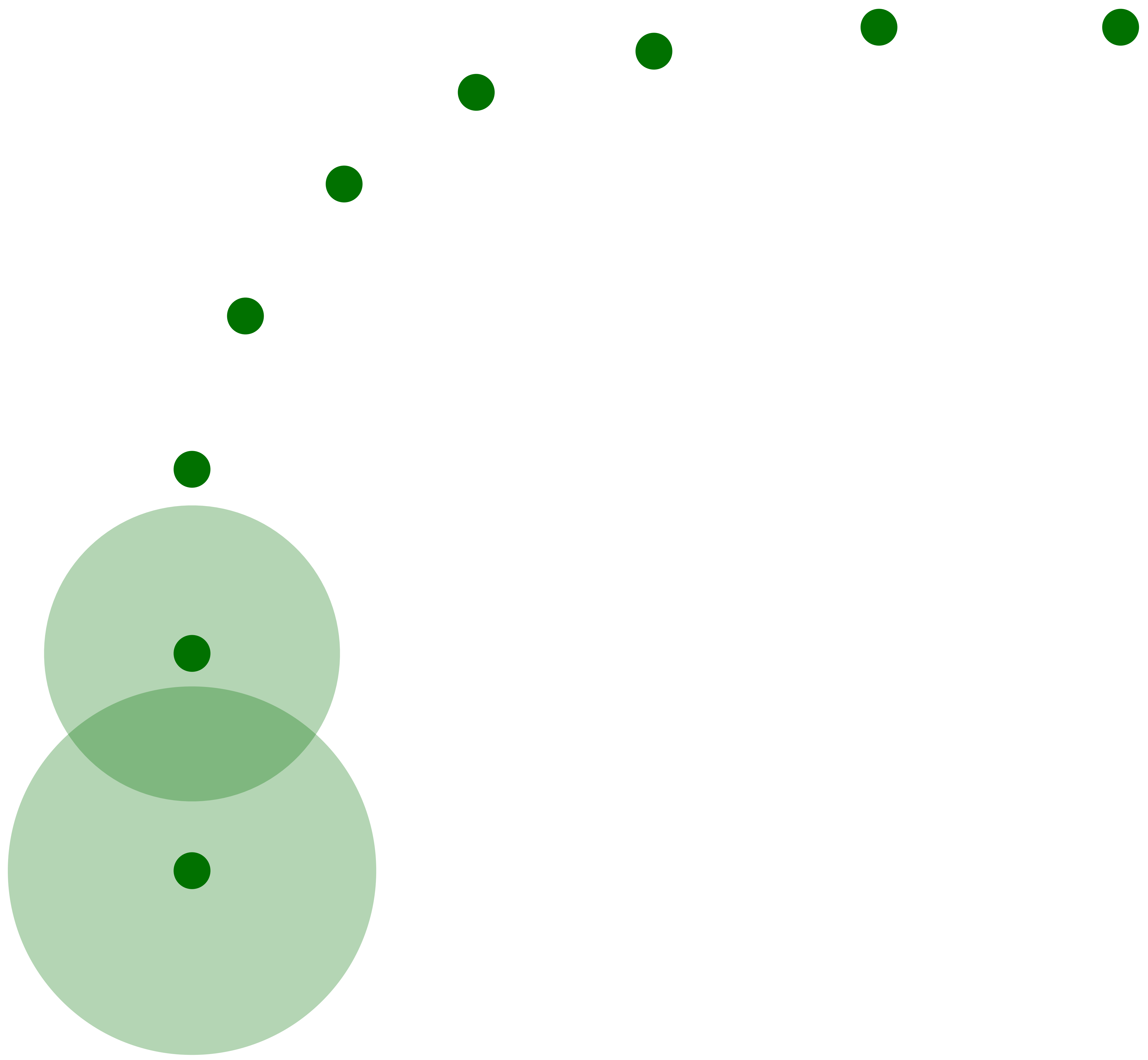
With probability δ

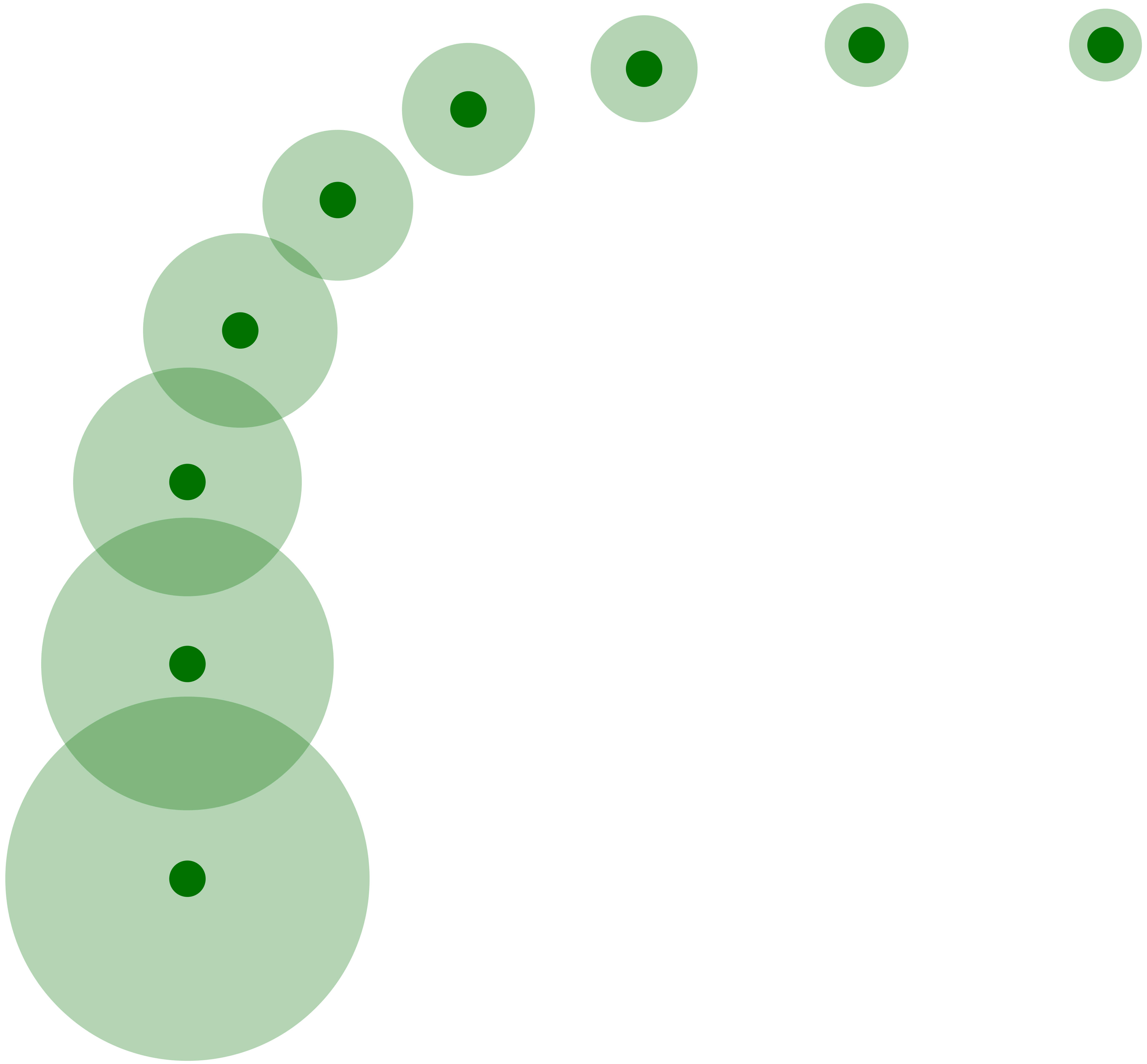

$\hat{f}(\overrightarrow{X}_t)$
$\hat{f}(\overrightarrow{X}_2)$
$\hat{f}(\overrightarrow{X}_1)$






Known





More Data…

…more certainty.

System MCs

Property $\mathbb{P}(r \mid q)$

System | some POMCs

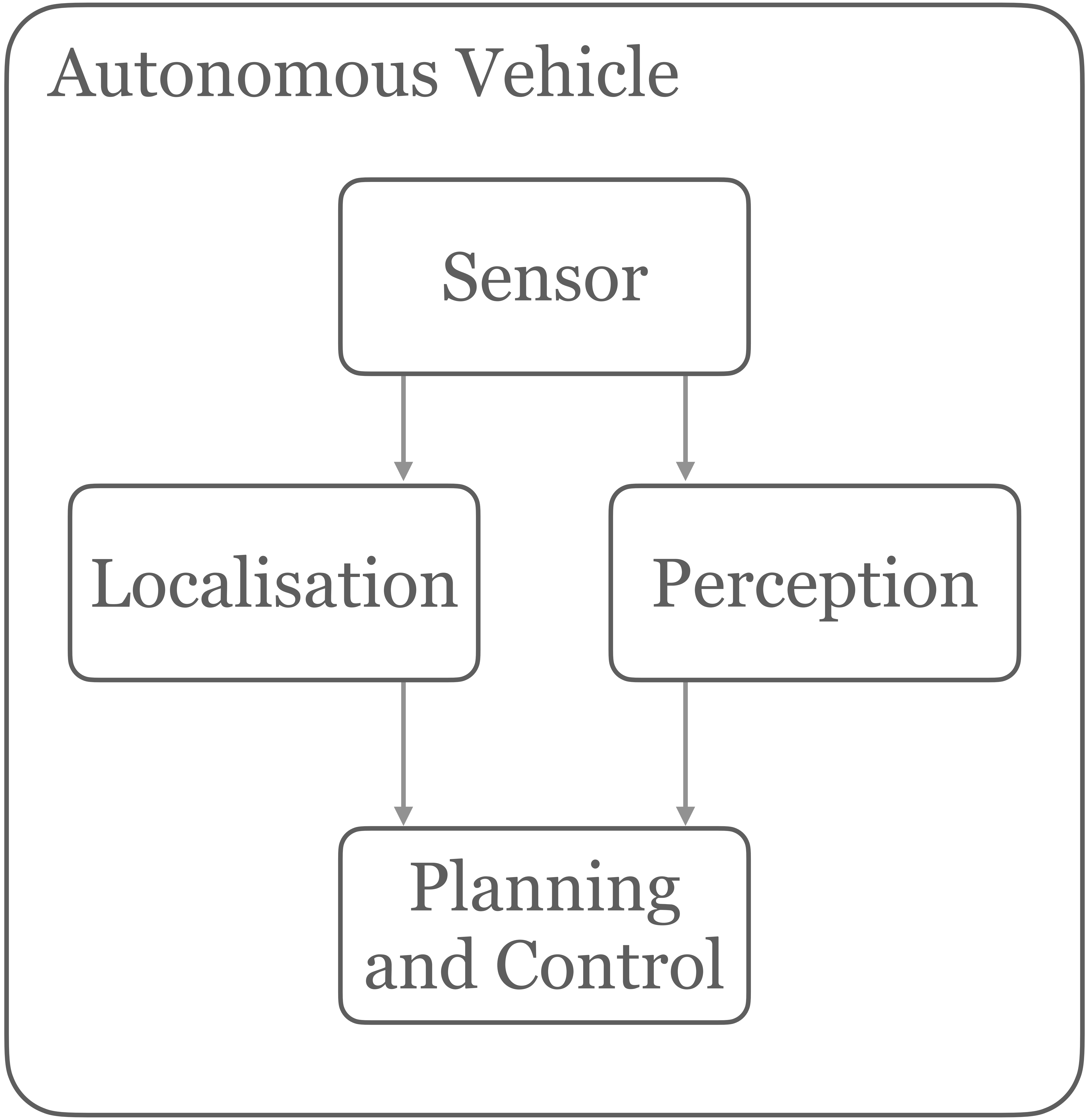
Property | $\mathbb{E}(f(X_{t:t+n}))$

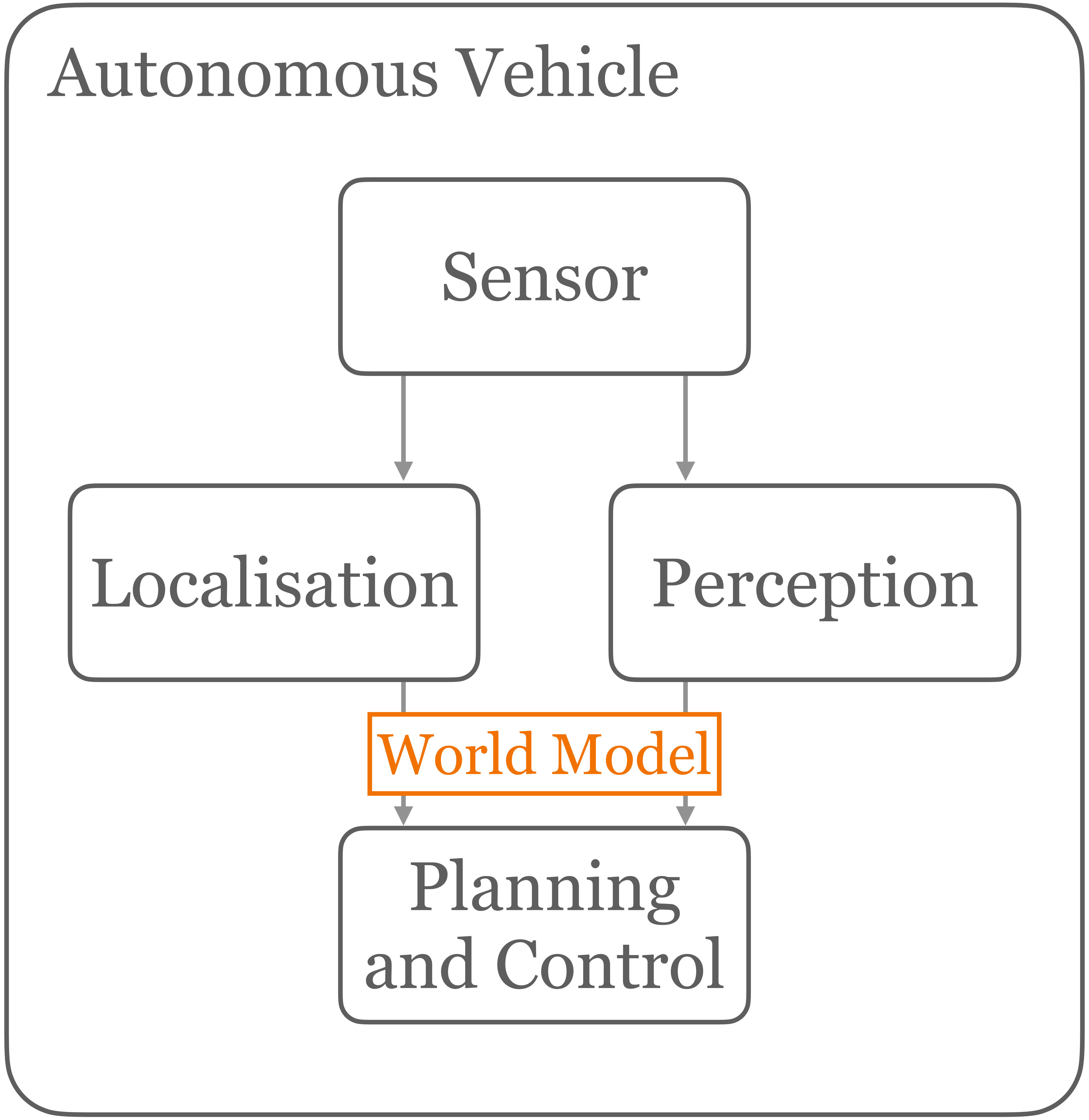
System $\mathbb{E}(X_{t+1} \mid \vec{x}_t) = \mathbb{E}(X_t \mid \vec{x}_{t-1}) + \Delta(x_t)$

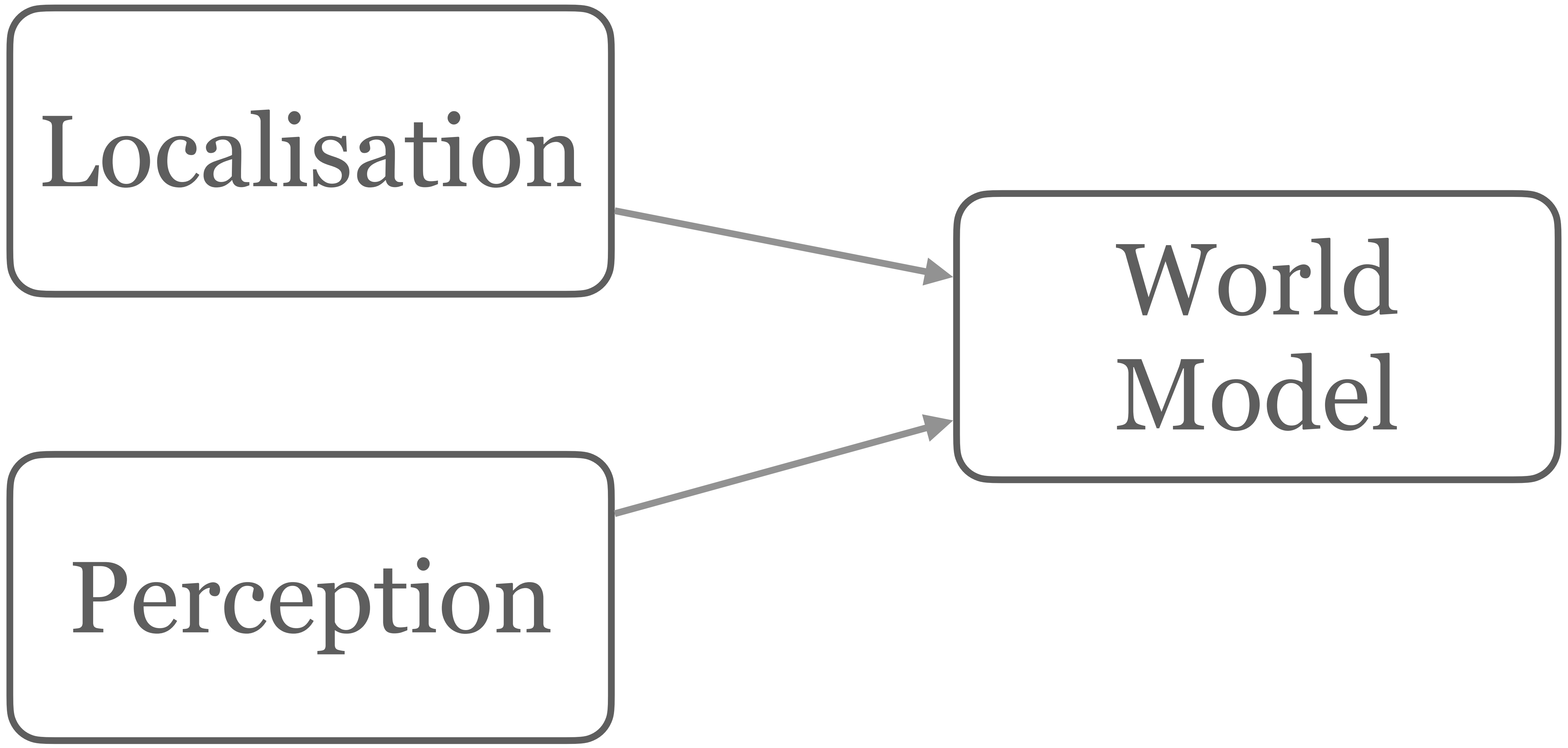
Property $\mathbb{E}(f(X_t) \mid \vec{x}_{t-1})$

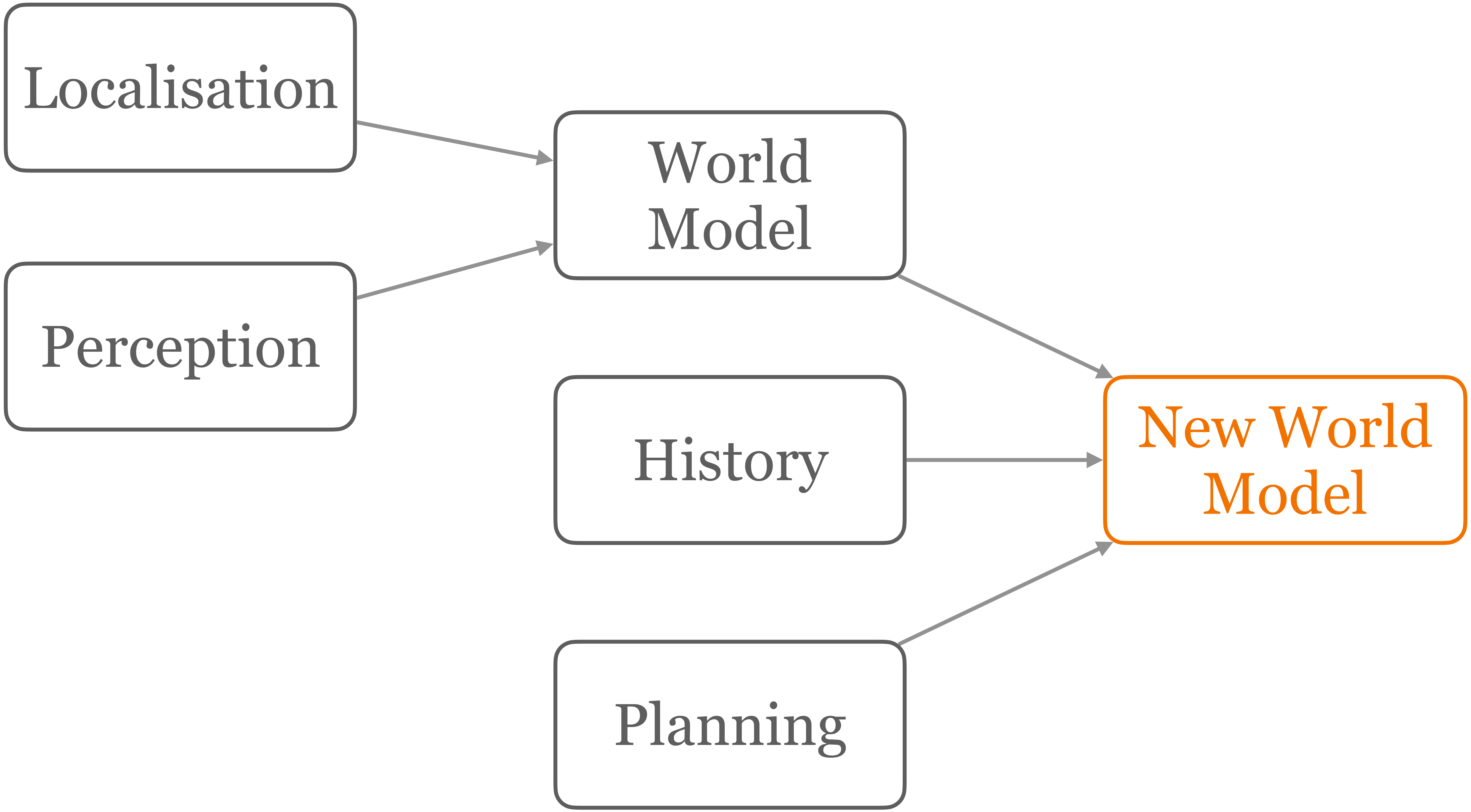
Possible Direction.

Use statistical monitoring to quantify and reduce the uncertainty in the world model.


Autonomous Vehicle
Sensor
Localisation
Perception
Planning and Control


Autonomous Vehicle
Sensor
Localisation
Perception
World Model
Planning and Control


Localisation
Perception
World
Model


Localisation
World Model
Perception
History
New World Model
Planning

Efficient Planing…

…in Stochastic Systems.

$$\overrightarrow{X} := \left(X_t\right)_{t>0}$$

a stochastic process

$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

a stochastic process

Environment

$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

a stochastic process

Agent

$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

a stochastic process

$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

a stochastic process

$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

a stochastic process

$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

a stochastic process

$$\gamma(\overrightarrow{X}_{t-1}, O_t) = A_t$$

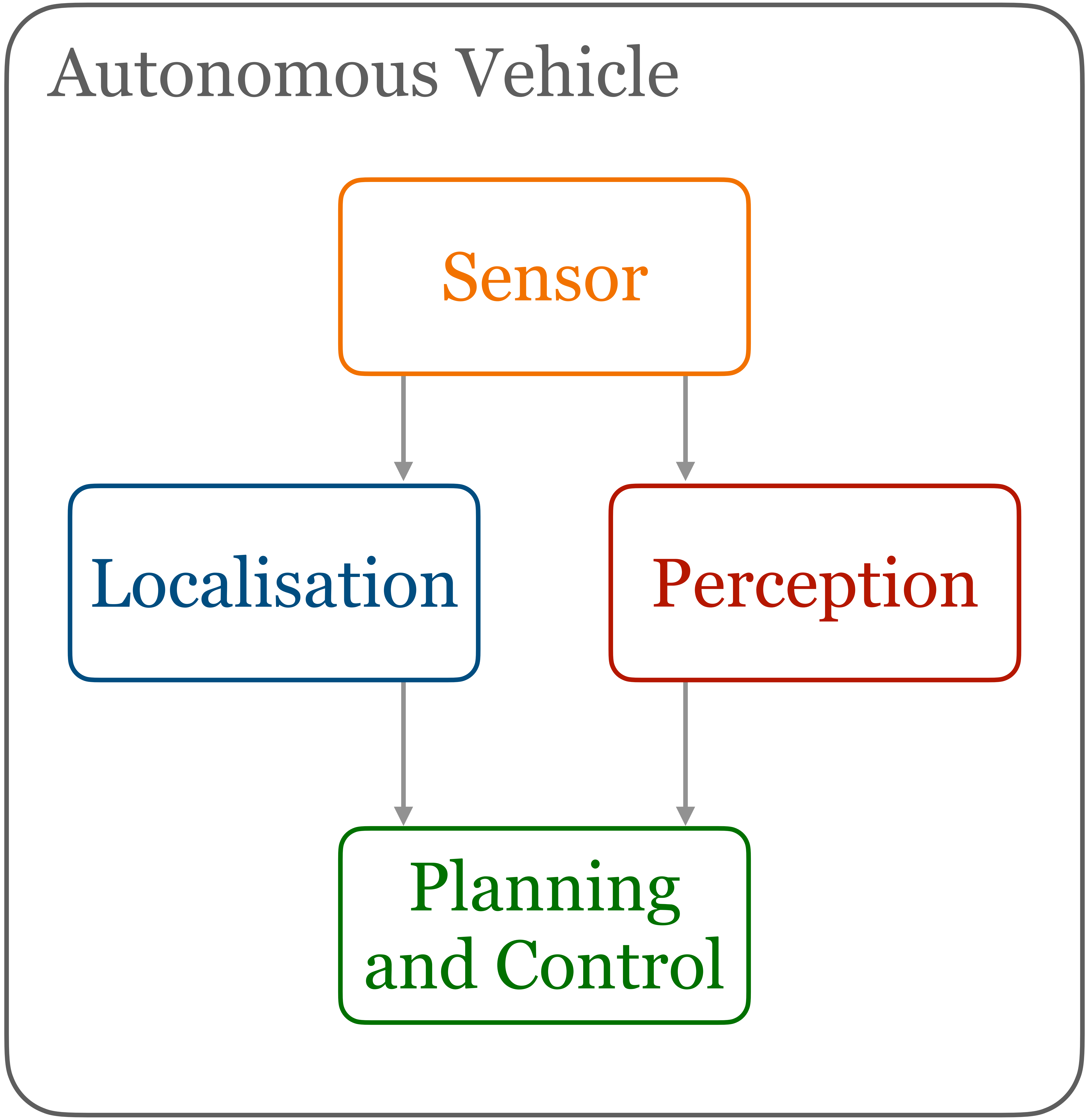
policy

$$\max_{\gamma} \mathbb{E}\gamma(\text{reward}(\overrightarrow{X}_n))$$

$$\text{s.t. } \mathbb{P}_{\gamma}(\text{cost}(\overrightarrow{X}_n)) = 1$$

Example.

Process starvation.


Autonomous Vehicle
Sensor
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O_t ... world model

A_t ... allocation of one unit of computation

$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

Sensor

Localisation

Perception

Planning and Control

O_t ... world model

A_t ... allocation of one unit of computation

Objective: Maximise utility

Utility

$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

O_t ... world model

A_t ... allocation of one unit of computation

Objective: Maximise utility

Constraint: Prevent Starvation

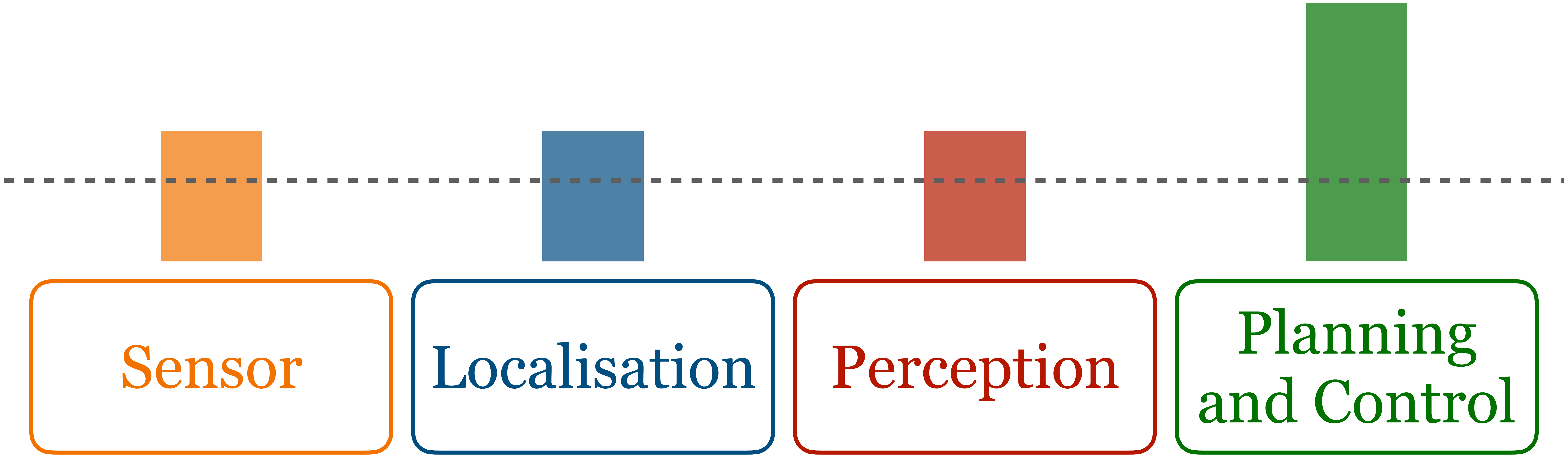
$$O_1, A_1, O_2, A_2, O_3, A_3, \ldots O_n, A_n$$

Sensor

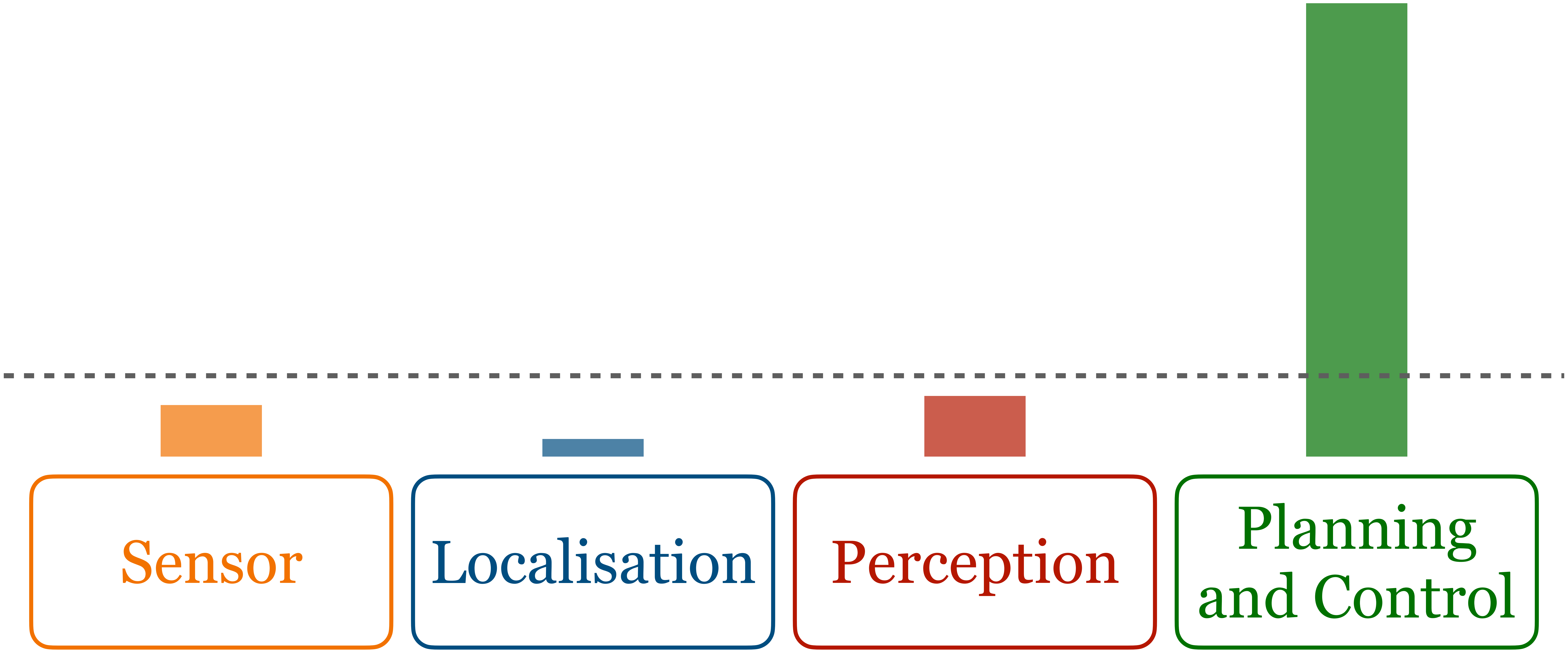
Localisation

Perception

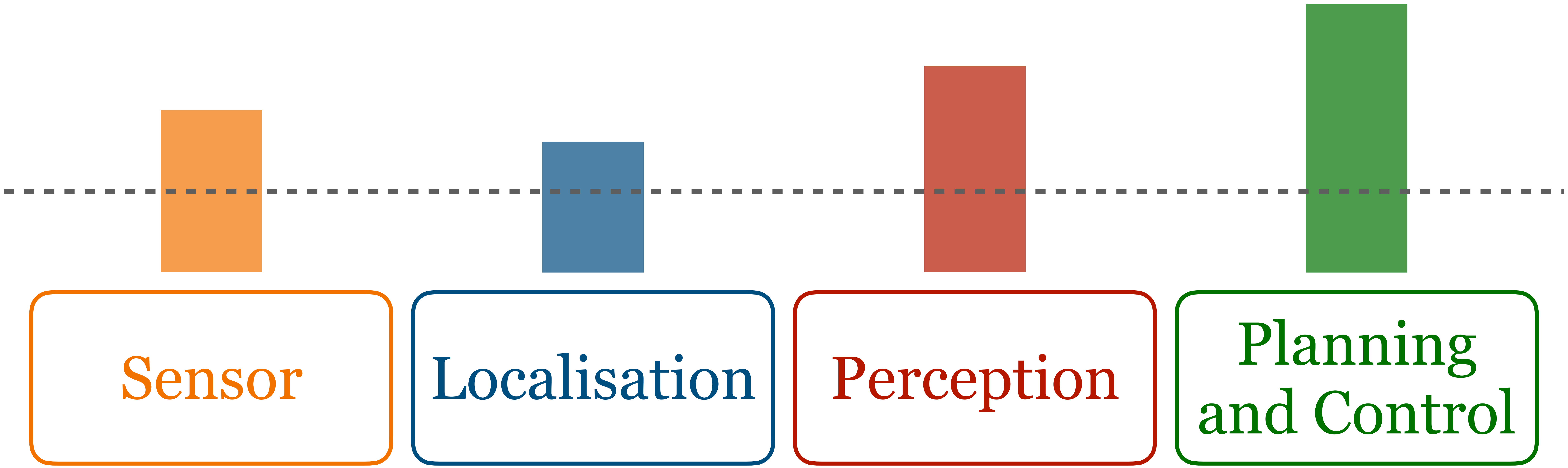
Planning and Control



$$\frac{1}{5}\sum_{t=1}^{5}\mathbf{1}[A_t = S] \qquad \frac{1}{5}\sum_{t=1}^{5}\mathbf{1}[A_t = L] \qquad \frac{1}{5}\sum_{t=1}^{5}\mathbf{1}[A_t = P] \qquad \frac{1}{5}\sum_{t=1}^{5}\mathbf{1}[A_t = C]$$



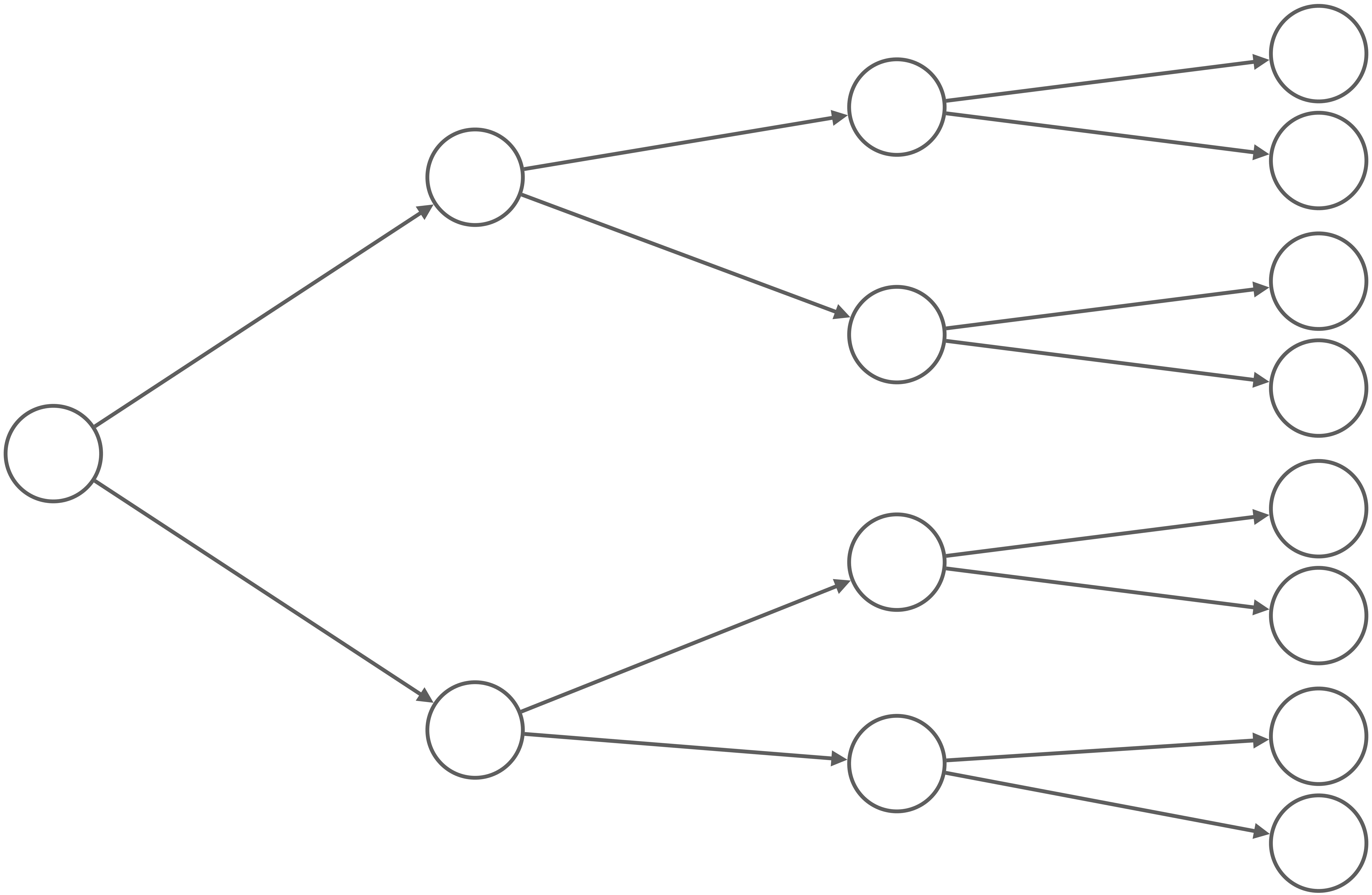
$$\frac{1}{k}\sum_{t=1}^{k}\mathbf{1}[A_t = S] \qquad \frac{1}{k}\sum_{t=1}^{k}\mathbf{1}[A_t = L] \qquad \frac{1}{k}\sum_{t=1}^{k}\mathbf{1}[A_t = P] \qquad \frac{1}{k}\sum_{t=1}^{k}\mathbf{1}[A_t = C]$$



$\frac{1}{n}\sum_{t=1}^{n}\mathbf{1}[A_t = S]$ $\frac{1}{n}\sum_{t=1}^{n}\mathbf{1}[A_t = L]$ $\frac{1}{n}\sum_{t=1}^{n}\mathbf{1}[A_t = P]$ $\frac{1}{n}\sum_{t=1}^{n}\mathbf{1}[A_t = C]$

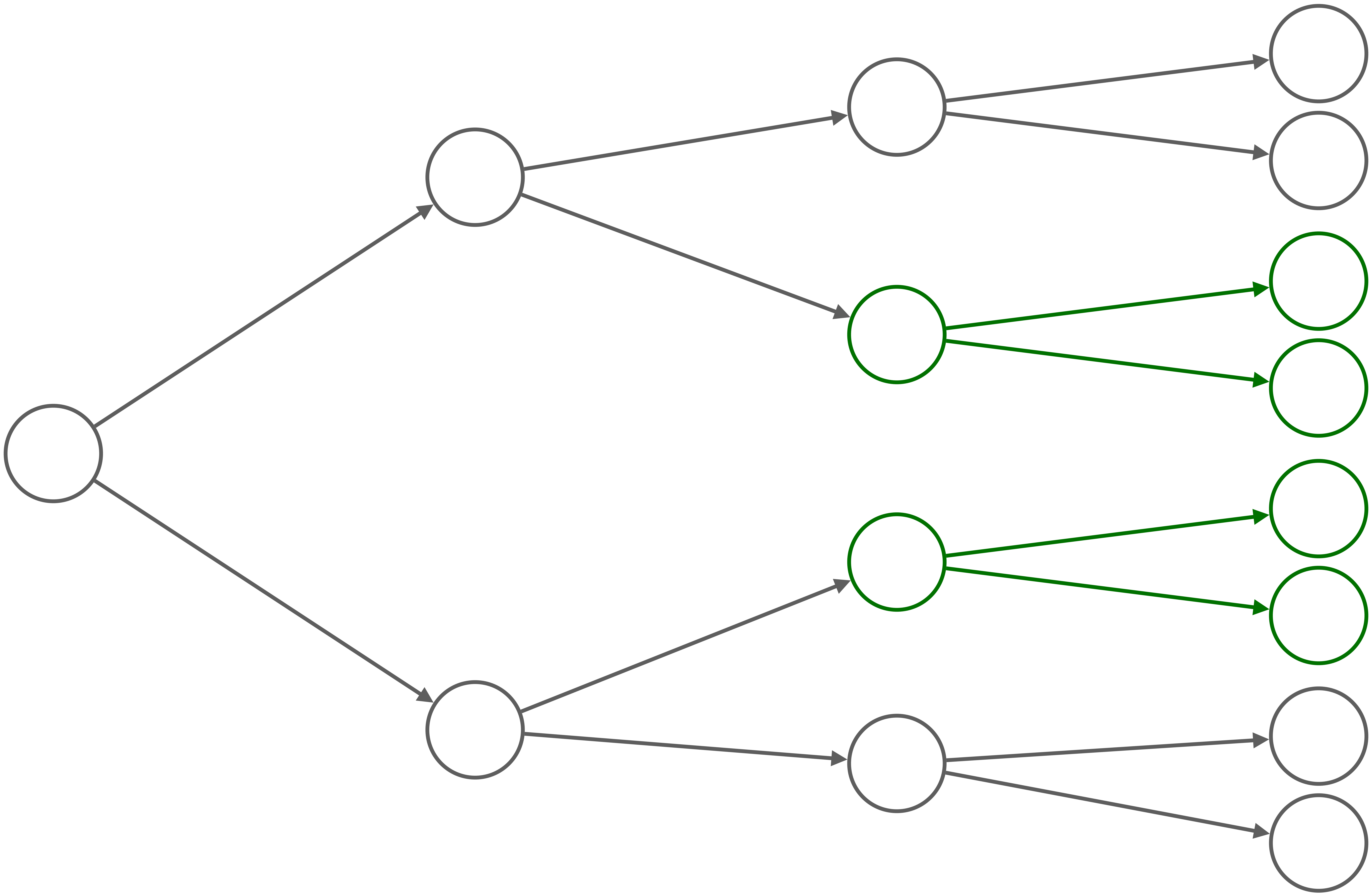
Problem.

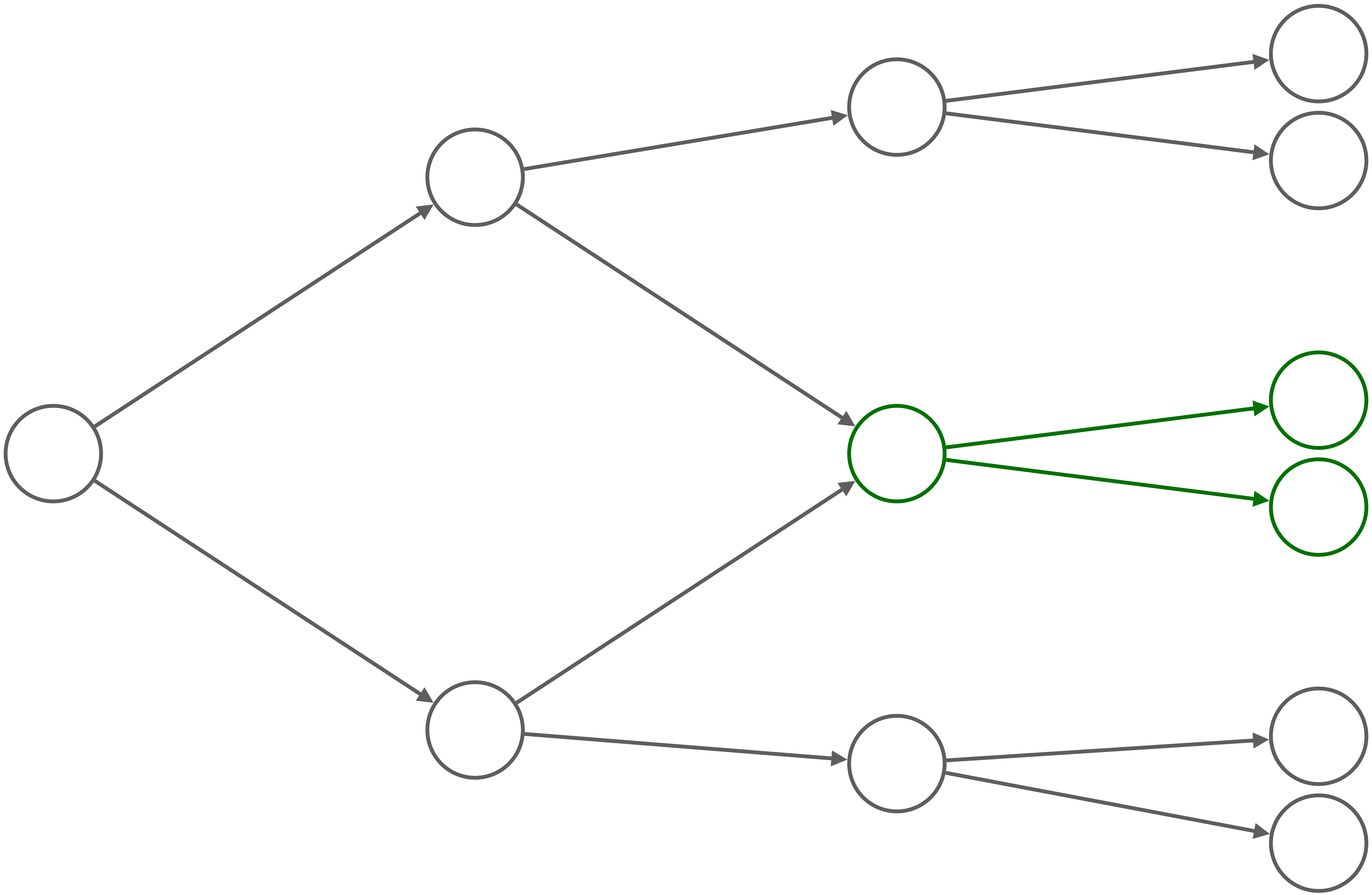
Exponential in time horizon.



Abstraction.

Sometimes different futures look the same.



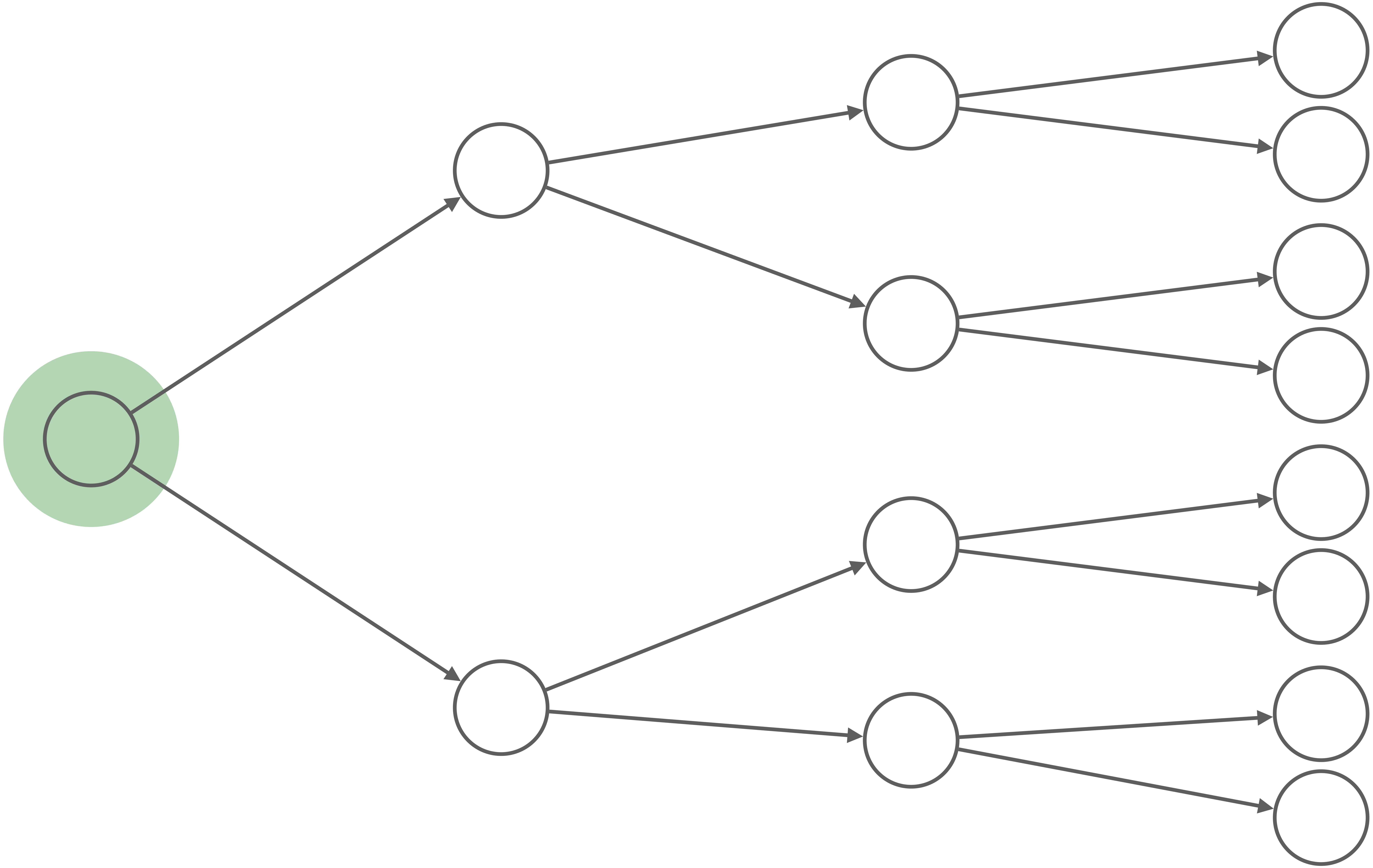


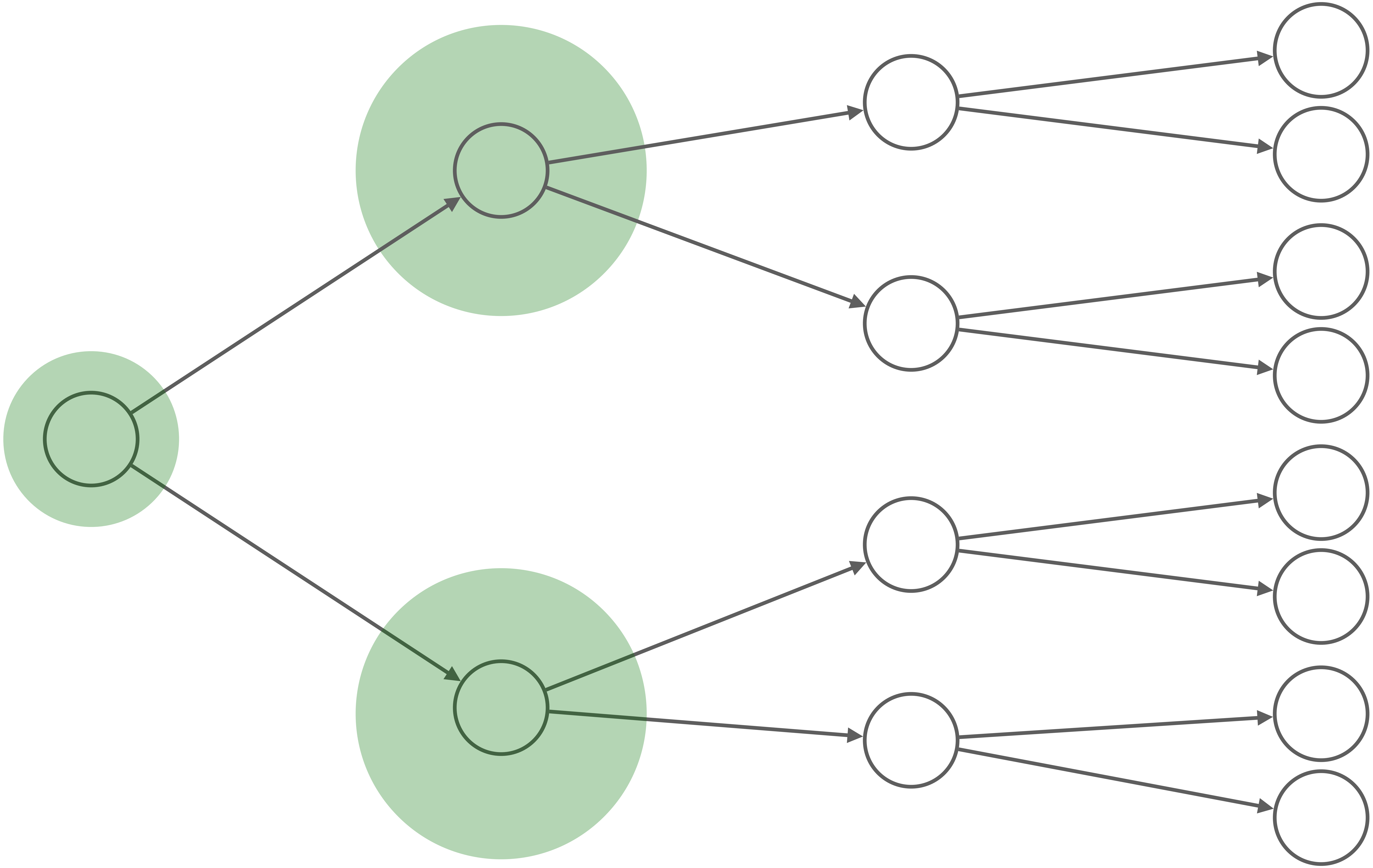
Future Projects

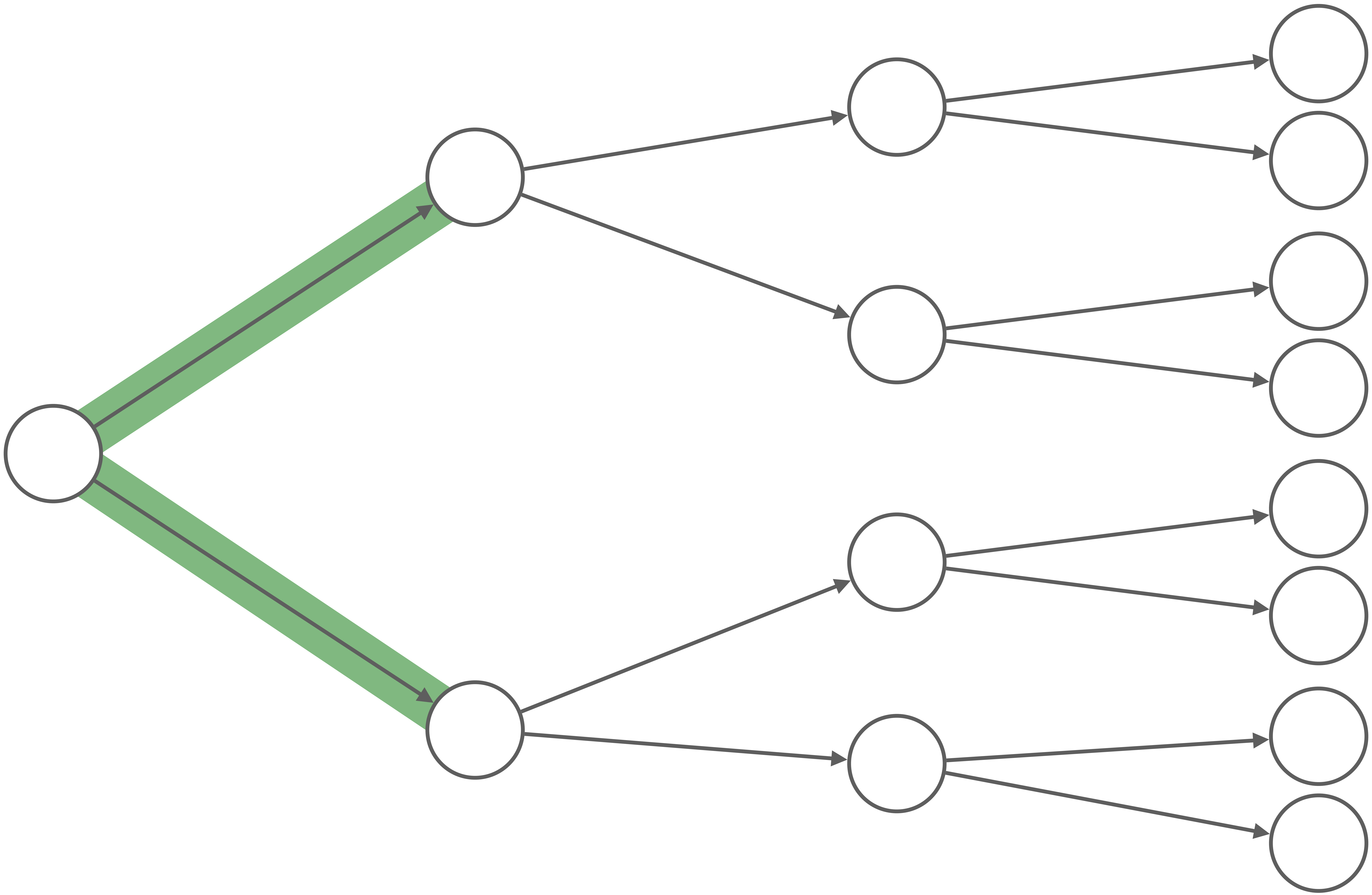
Two ongoing projects.

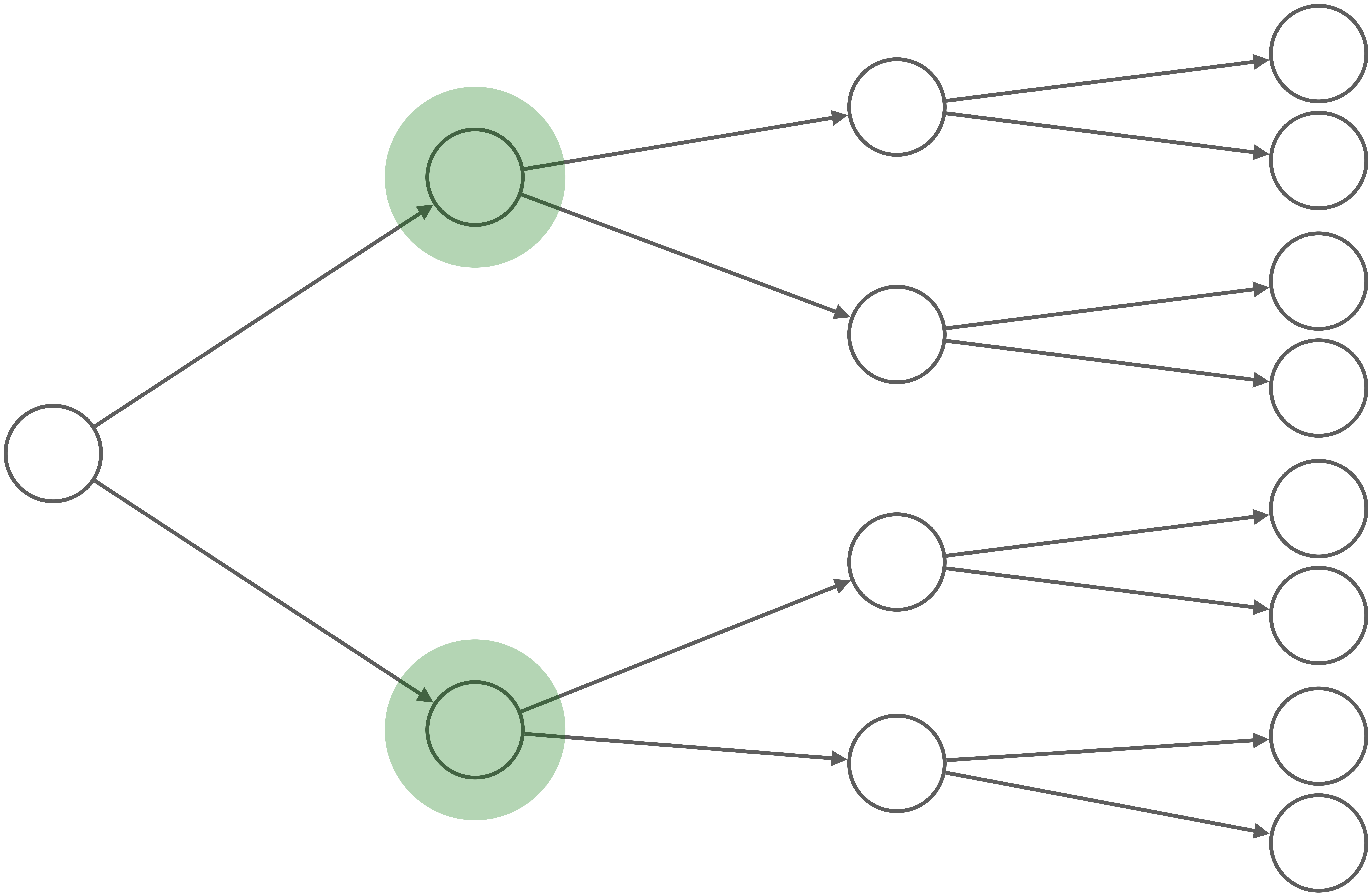
Uncertain Planning.

Combine statistical monitoring with efficient planning.



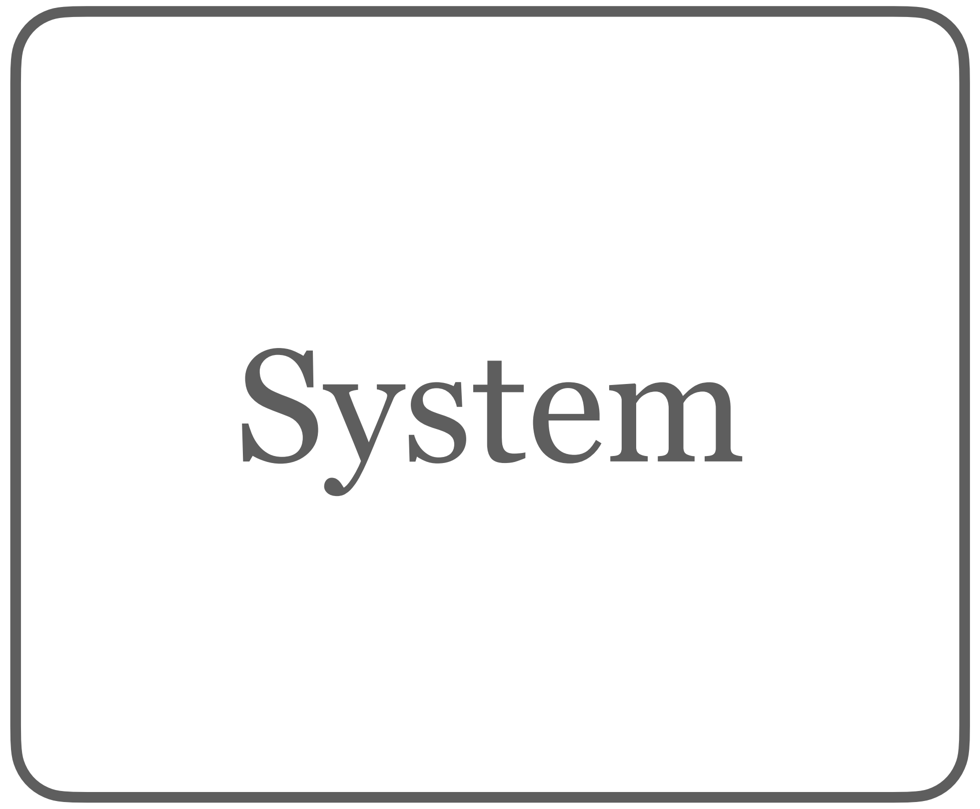


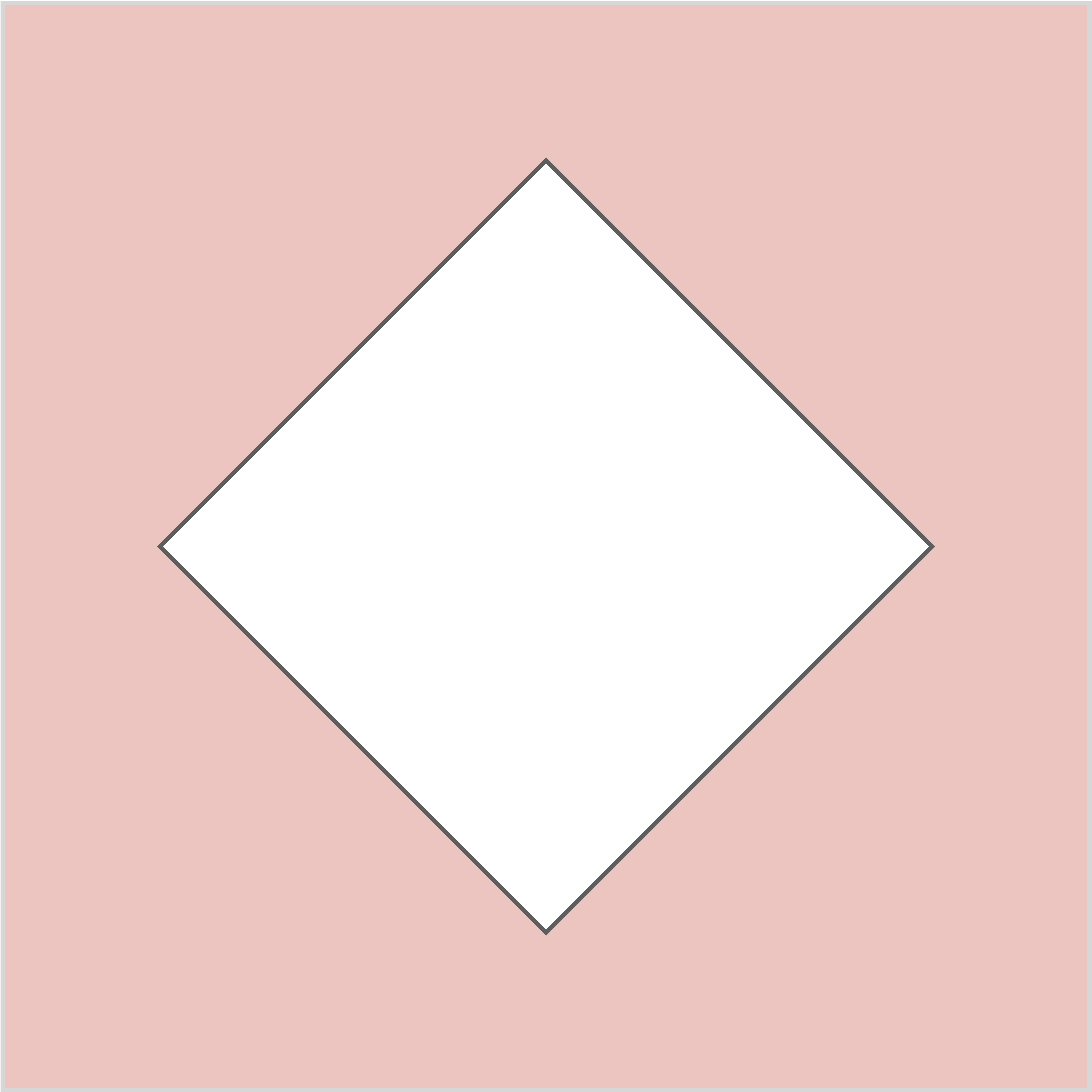


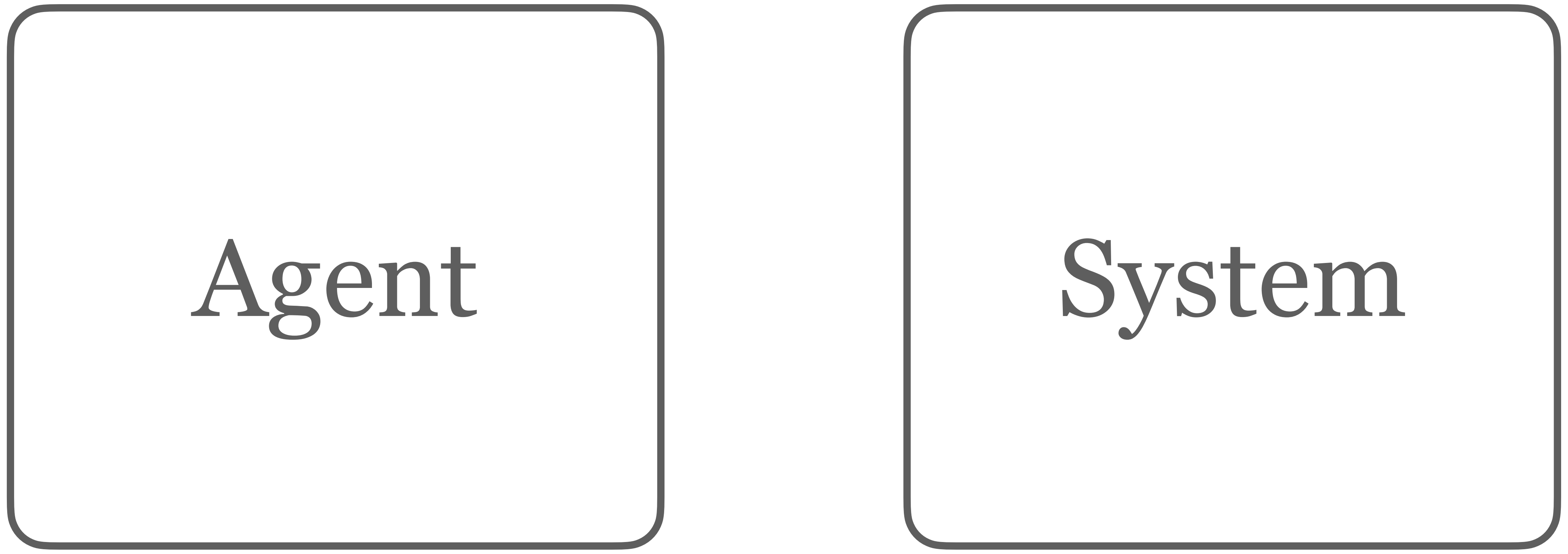


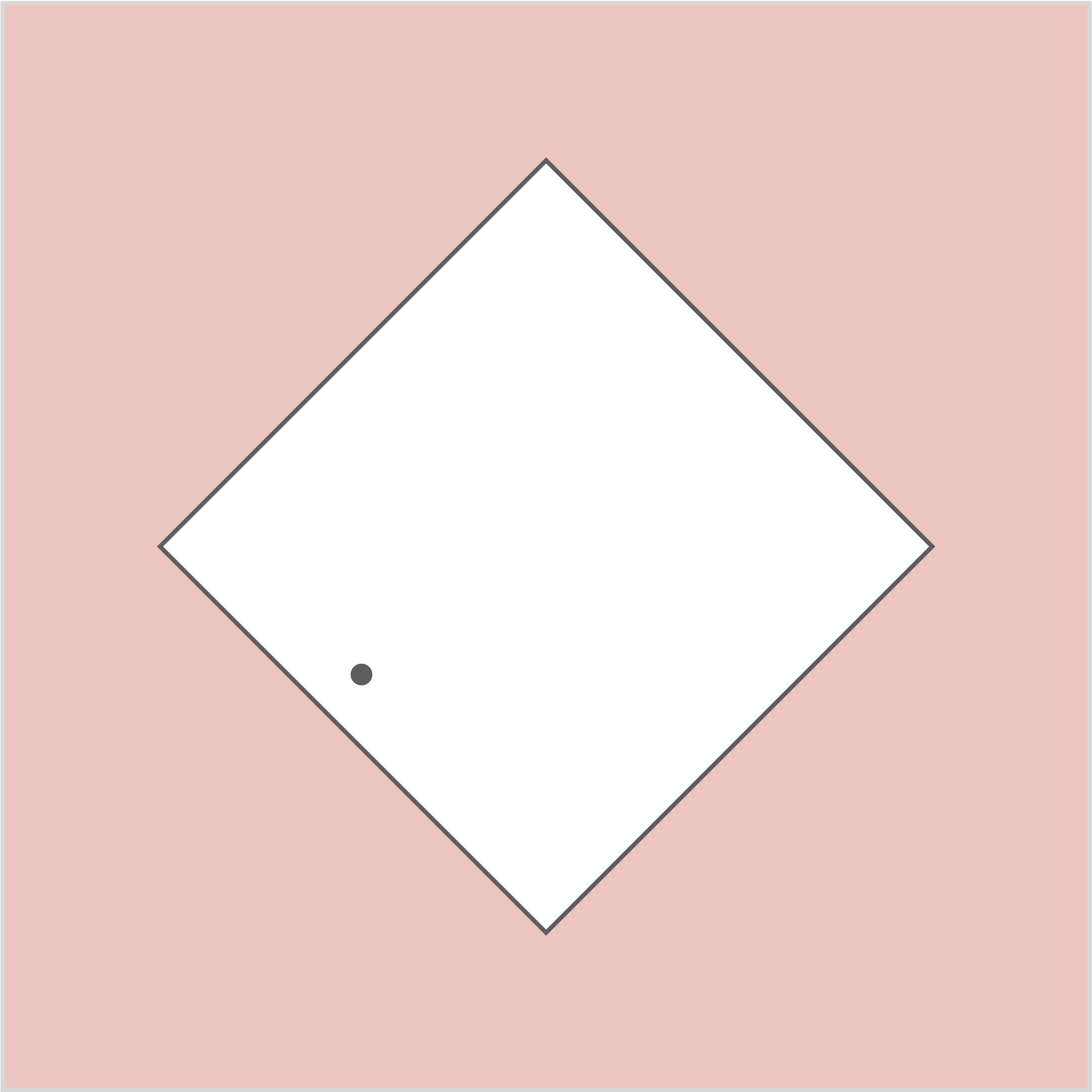
Neural-Certificates…

… and how to statistically verify them.


System



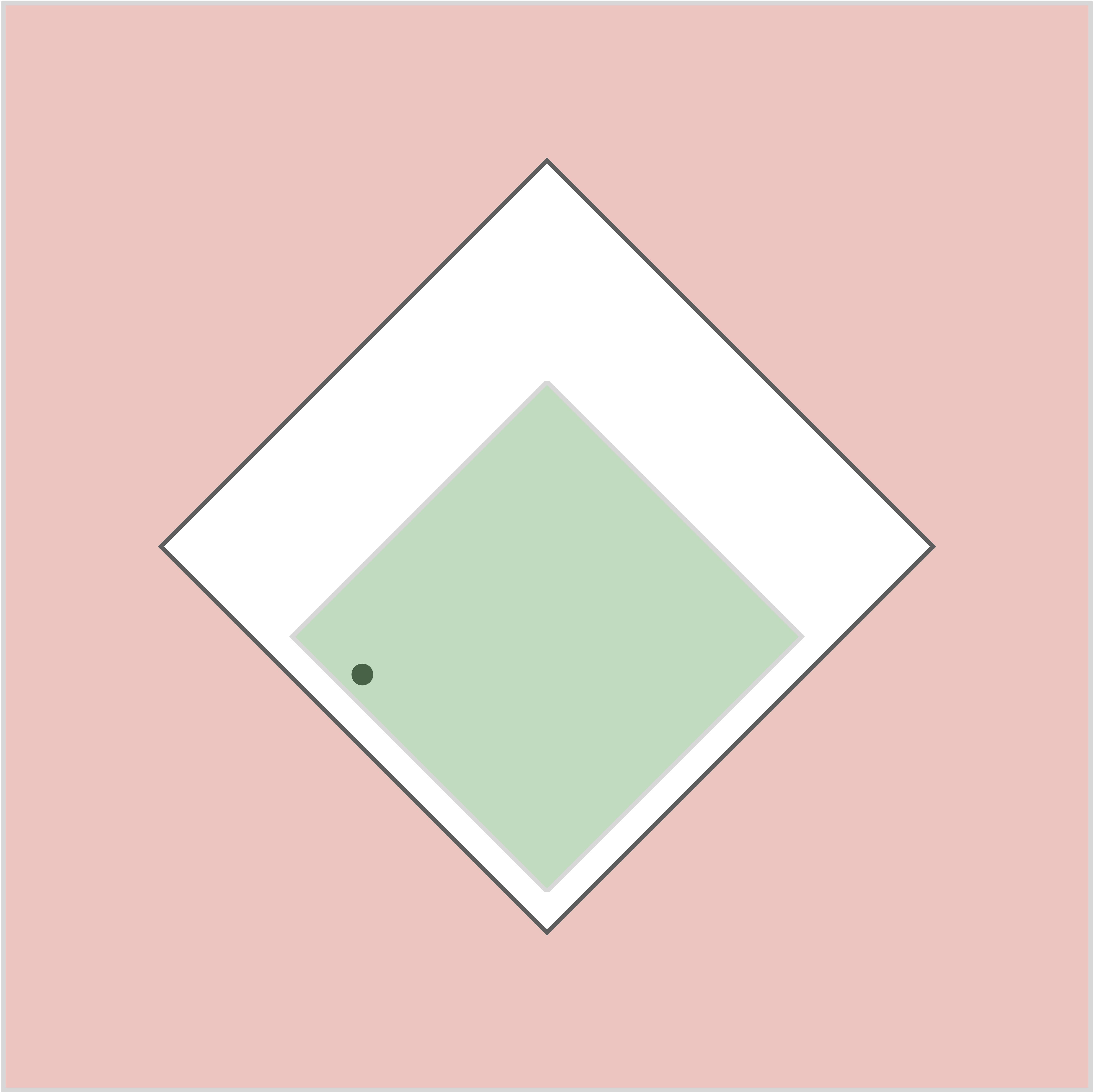

Agent
System



Certificate

Agent

System



Lyapunov Function

Controler

Dynamic System

Super-
Martingale

Neural
Network

Dynamic
Stochastic
System

Neural Certificate

Neural Network

Dynamic Stochastic System

Neural Certificate

Neural Network

Dynamic Stochastic System

*Is the **neural certificate** a proof of safety for the **neural controller** in the **stochastic system**?*

Neural Certificate

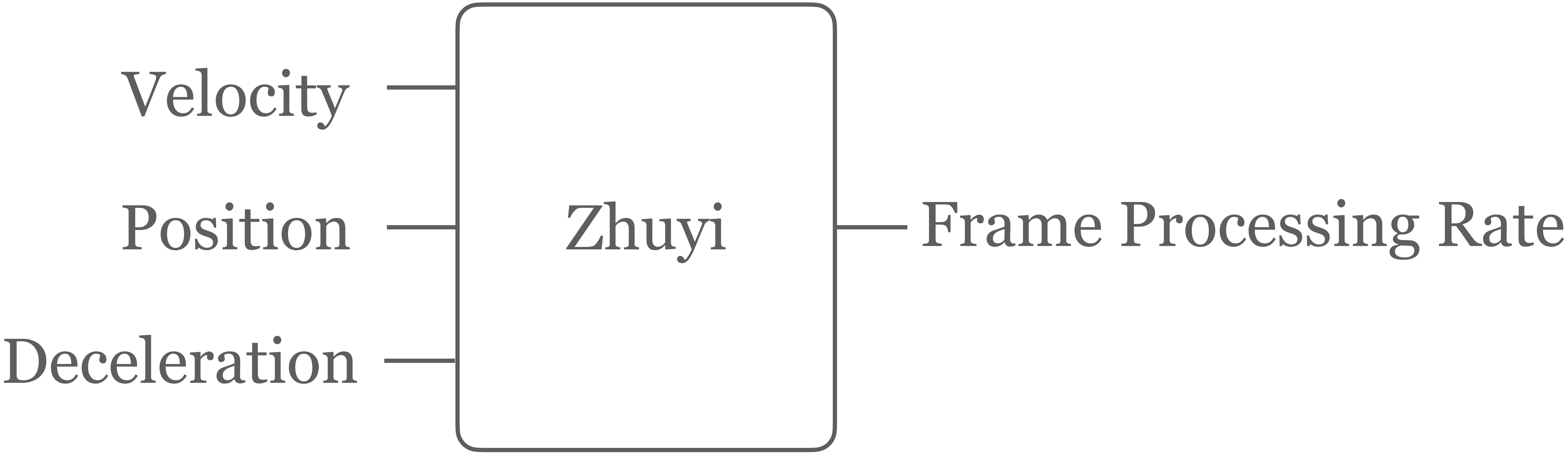
Neural Network

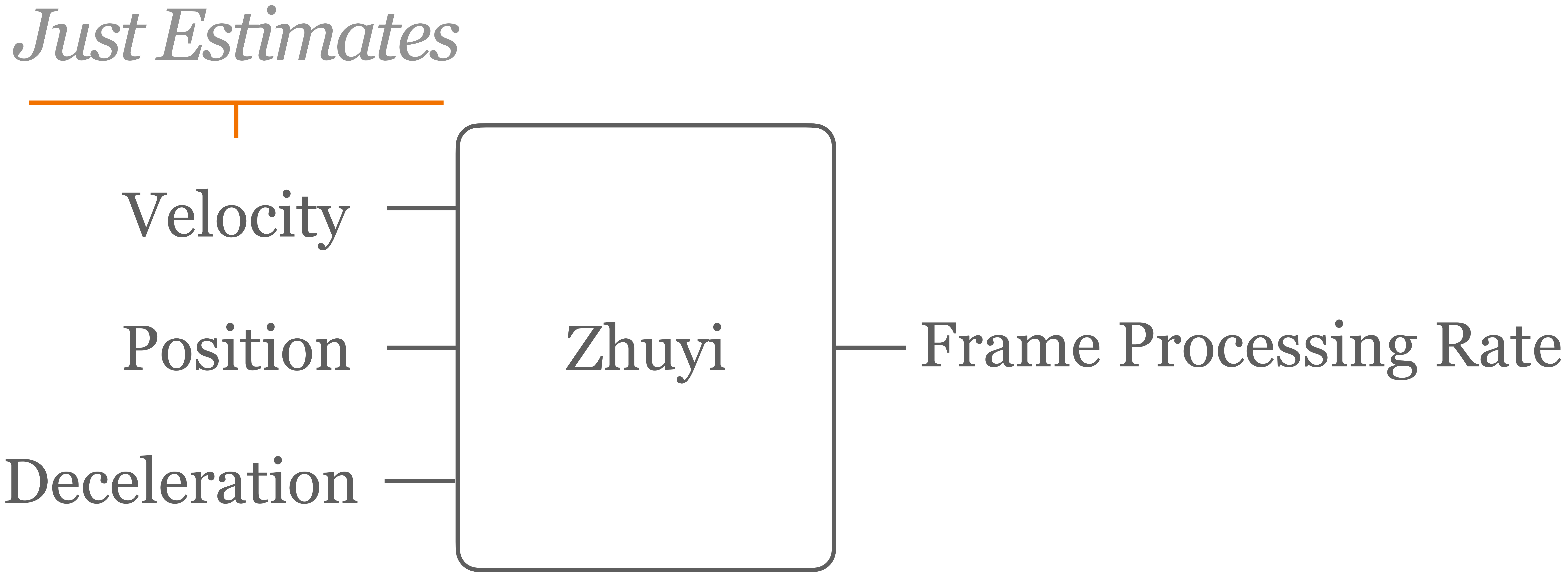
Dynamic Stochastic System

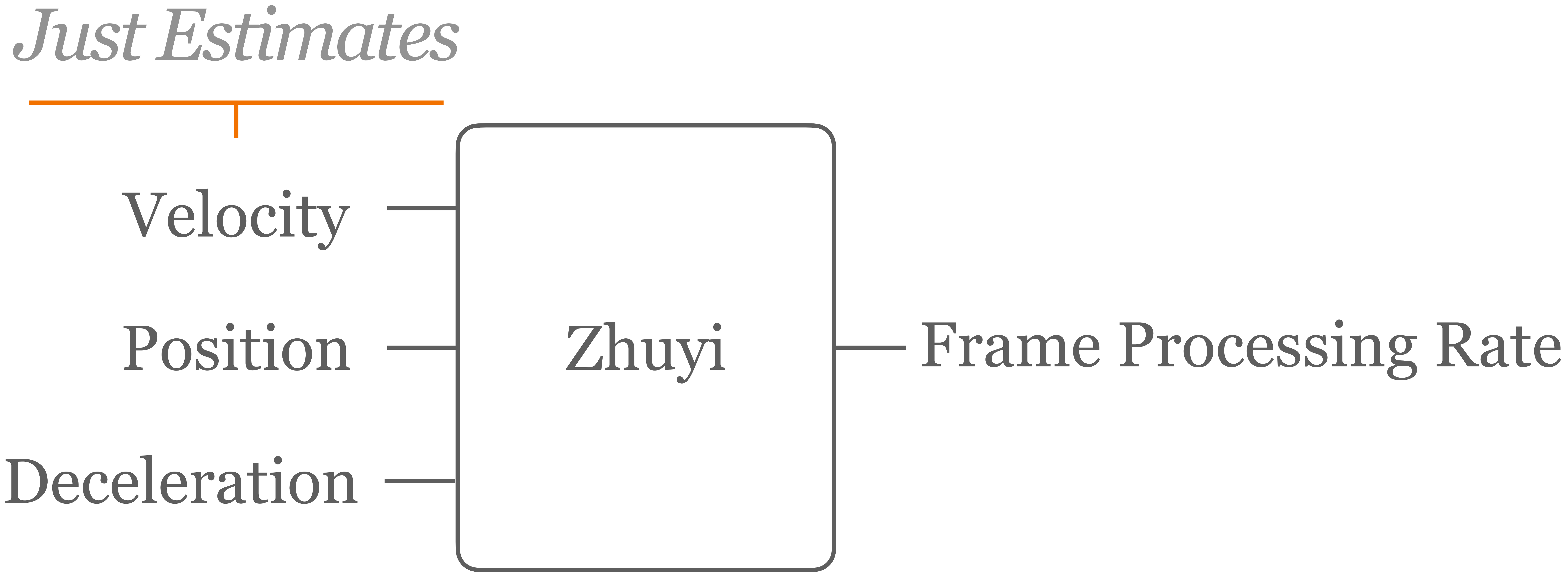
How can we ***verify*** *this statistically* ***without*** *knowing the system?*

Possible Direction.

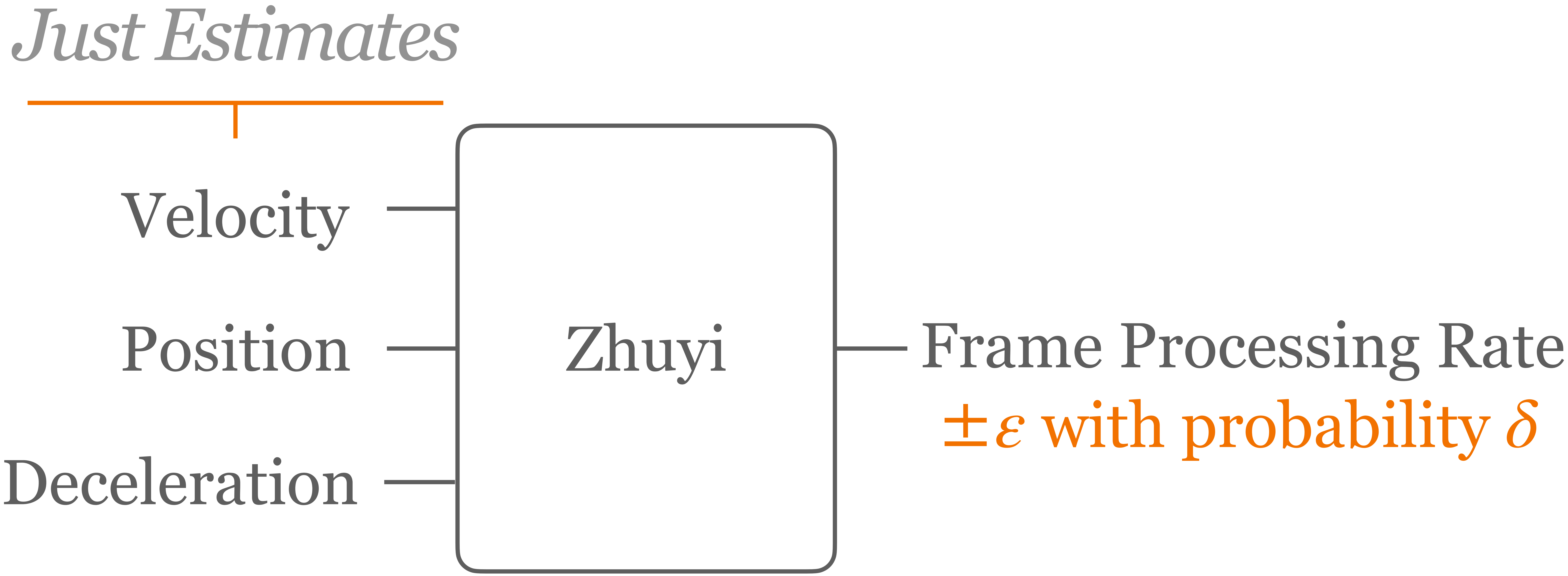
Improve safety in planning by quantifying state and/or model uncertainty.


Velocity
Position
Deceleration
Zhuyi
Frame Processing Rate


Just Estimates
Velocity
Position
Deceleration
Zhuyi
Frame Processing Rate



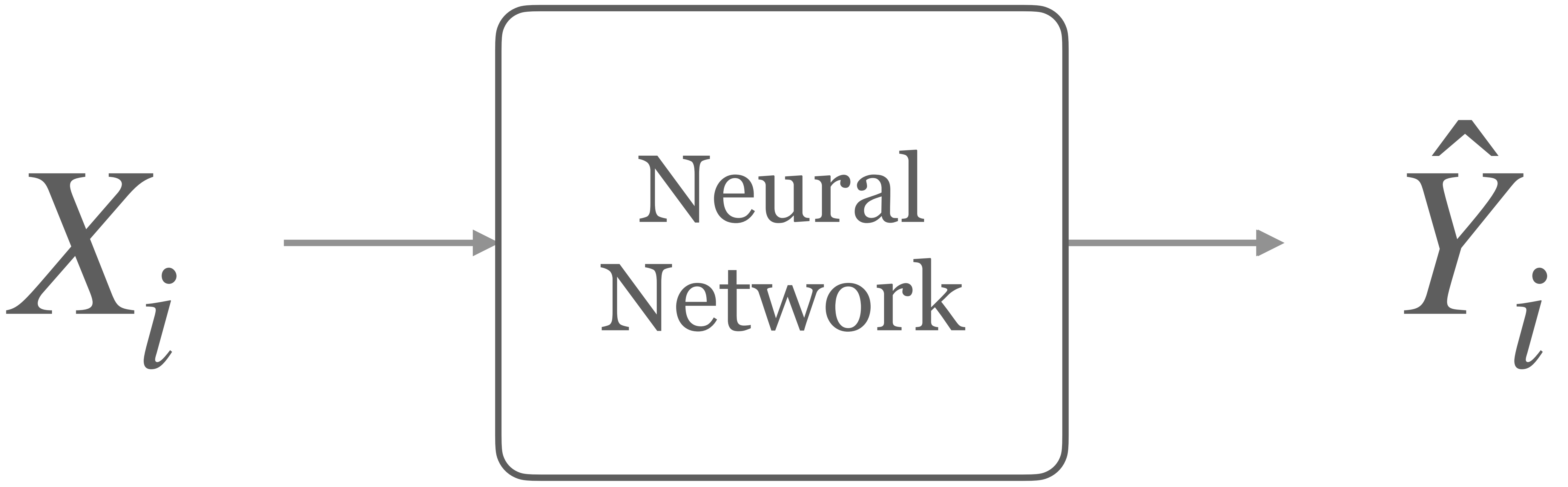
Carry uncertainty through the computation.

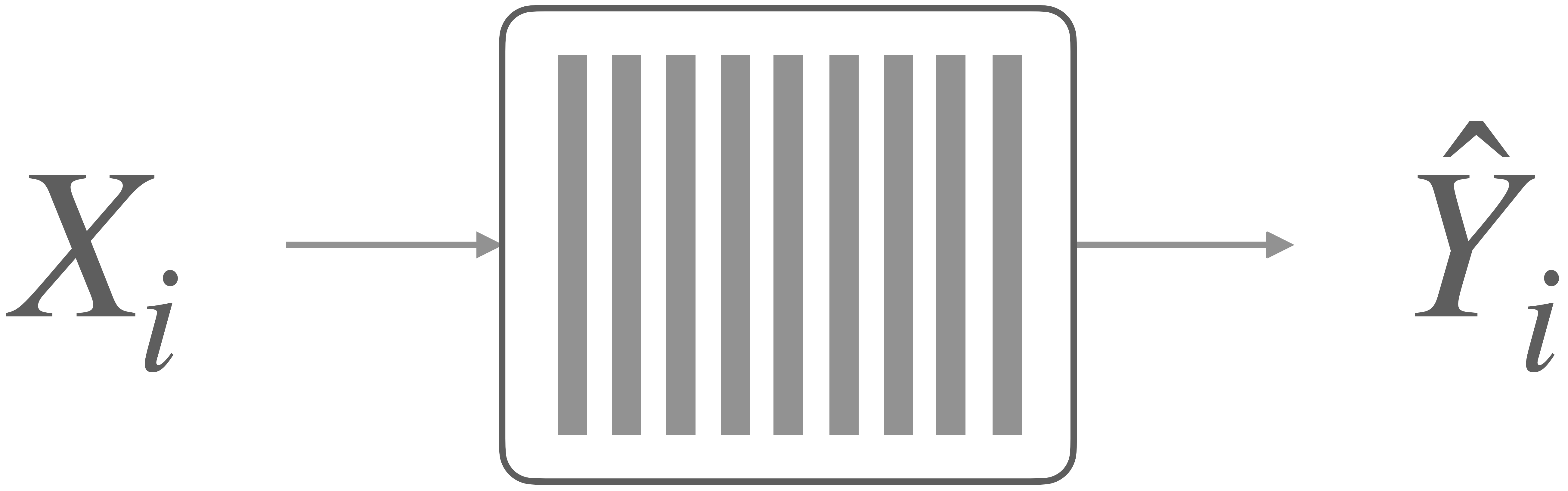


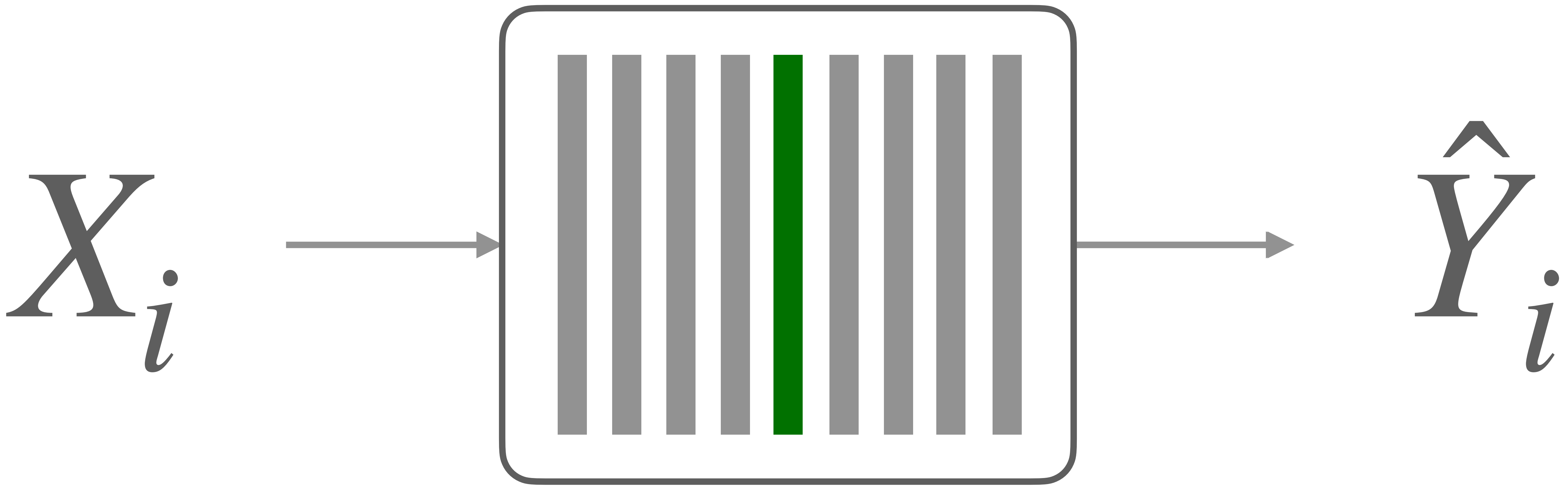
Carry uncertainty through the computation.

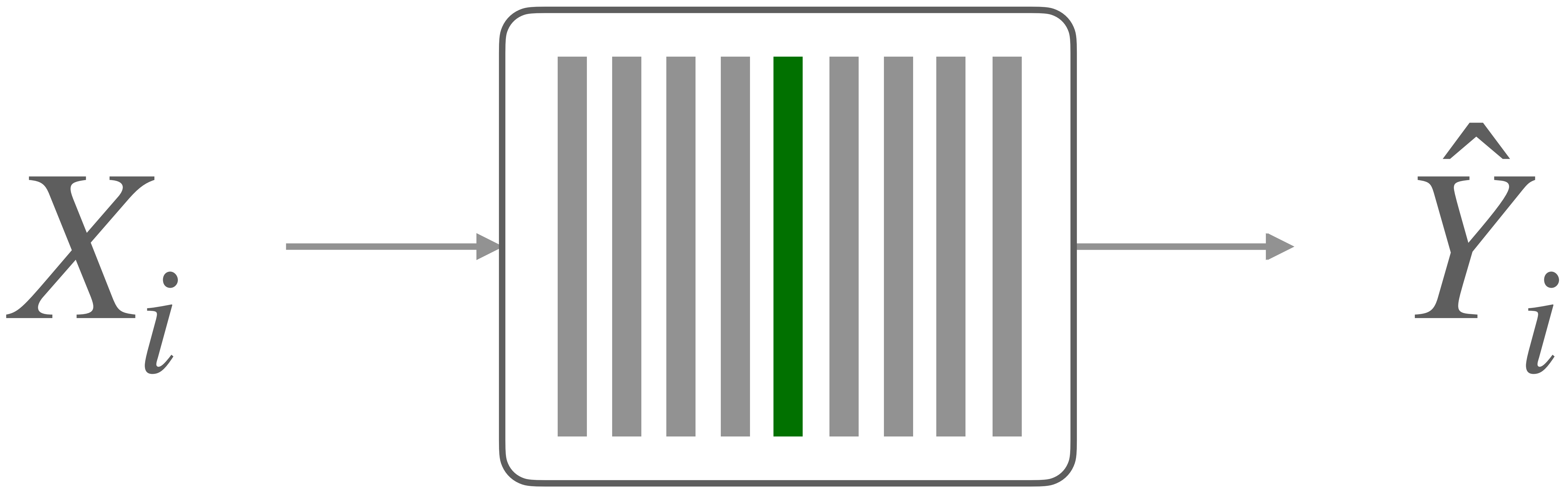
Novelty Detection.

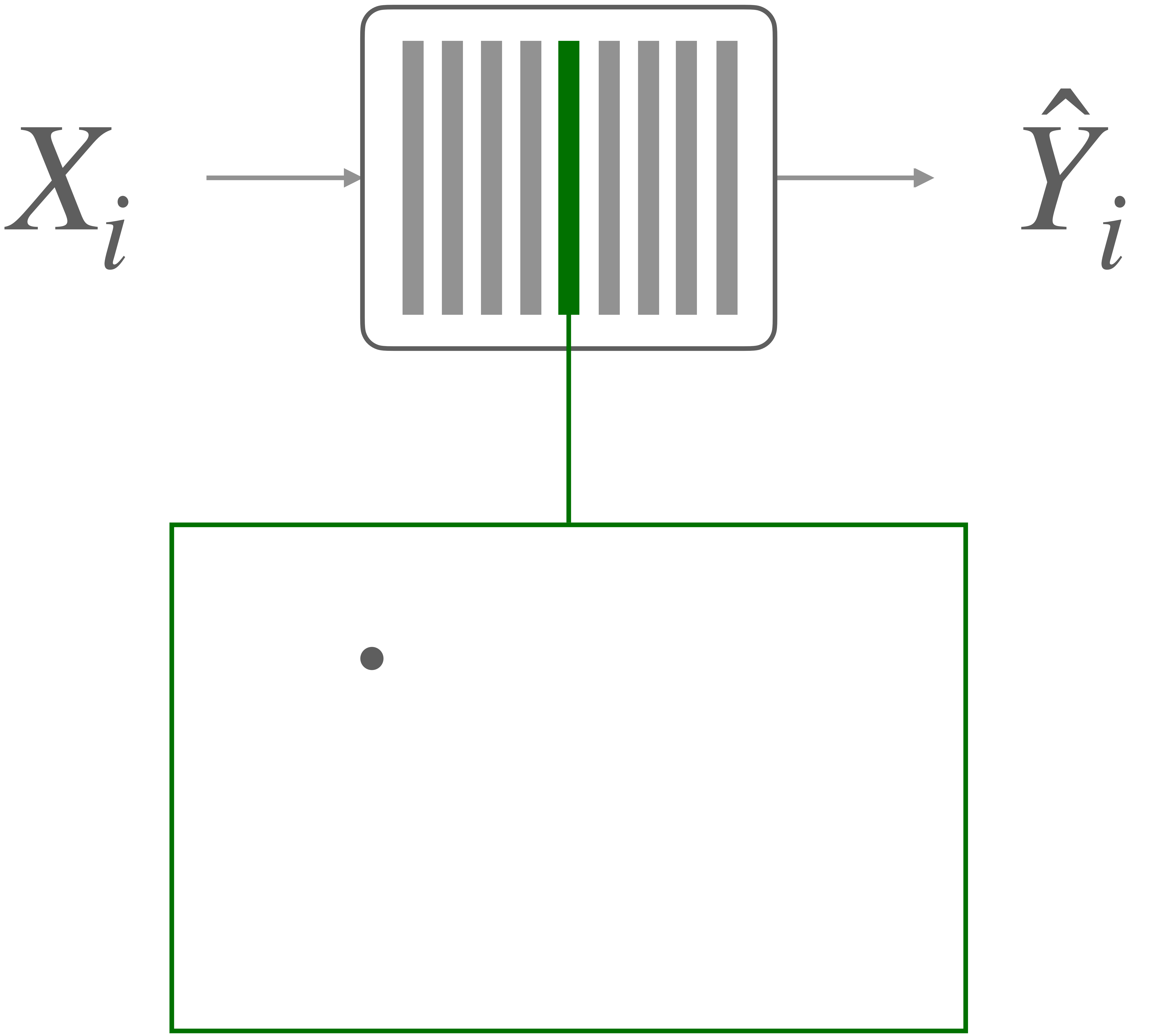
Single sample.

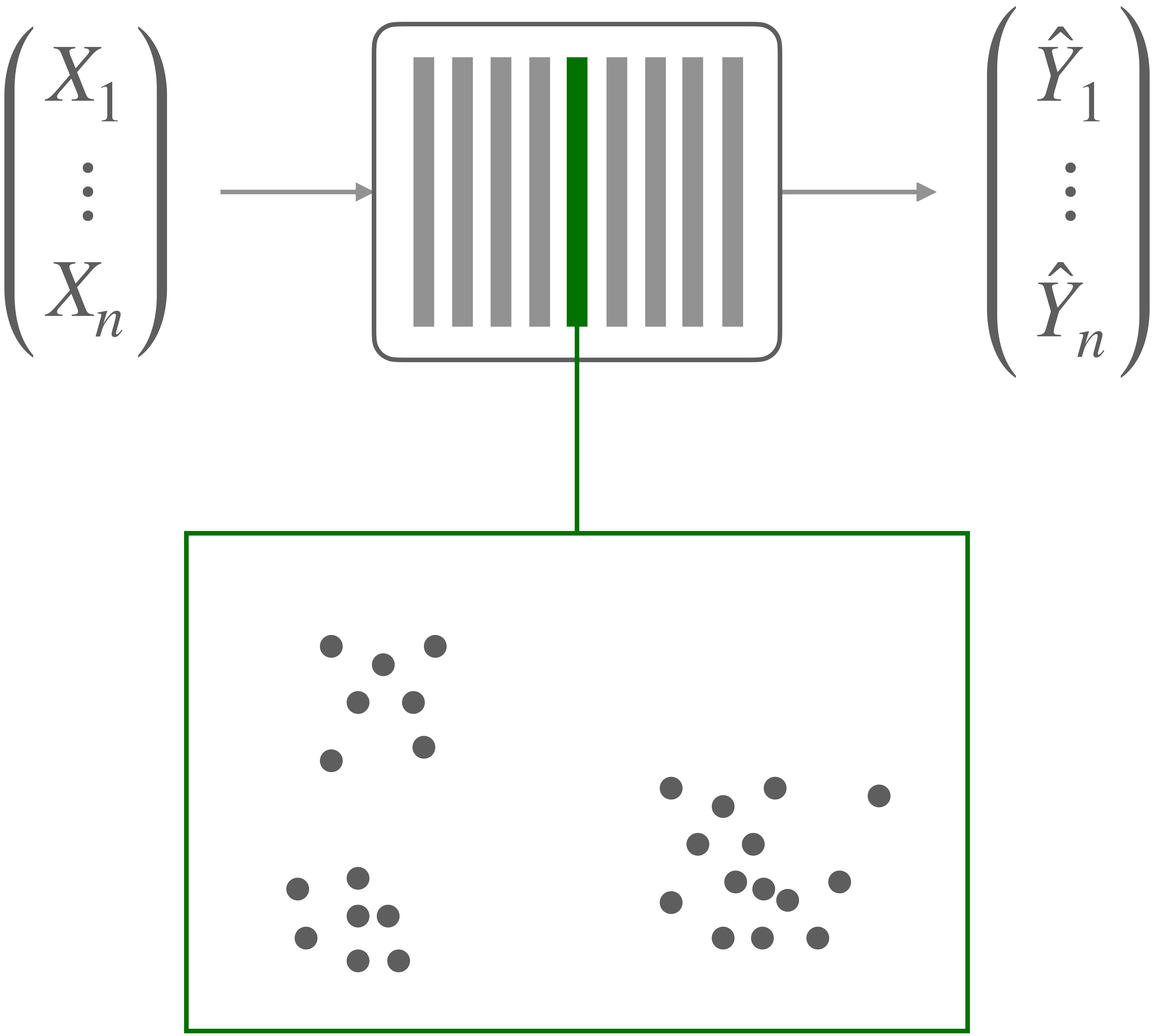

X_i
Neural Network
$\hat{Y}_i$

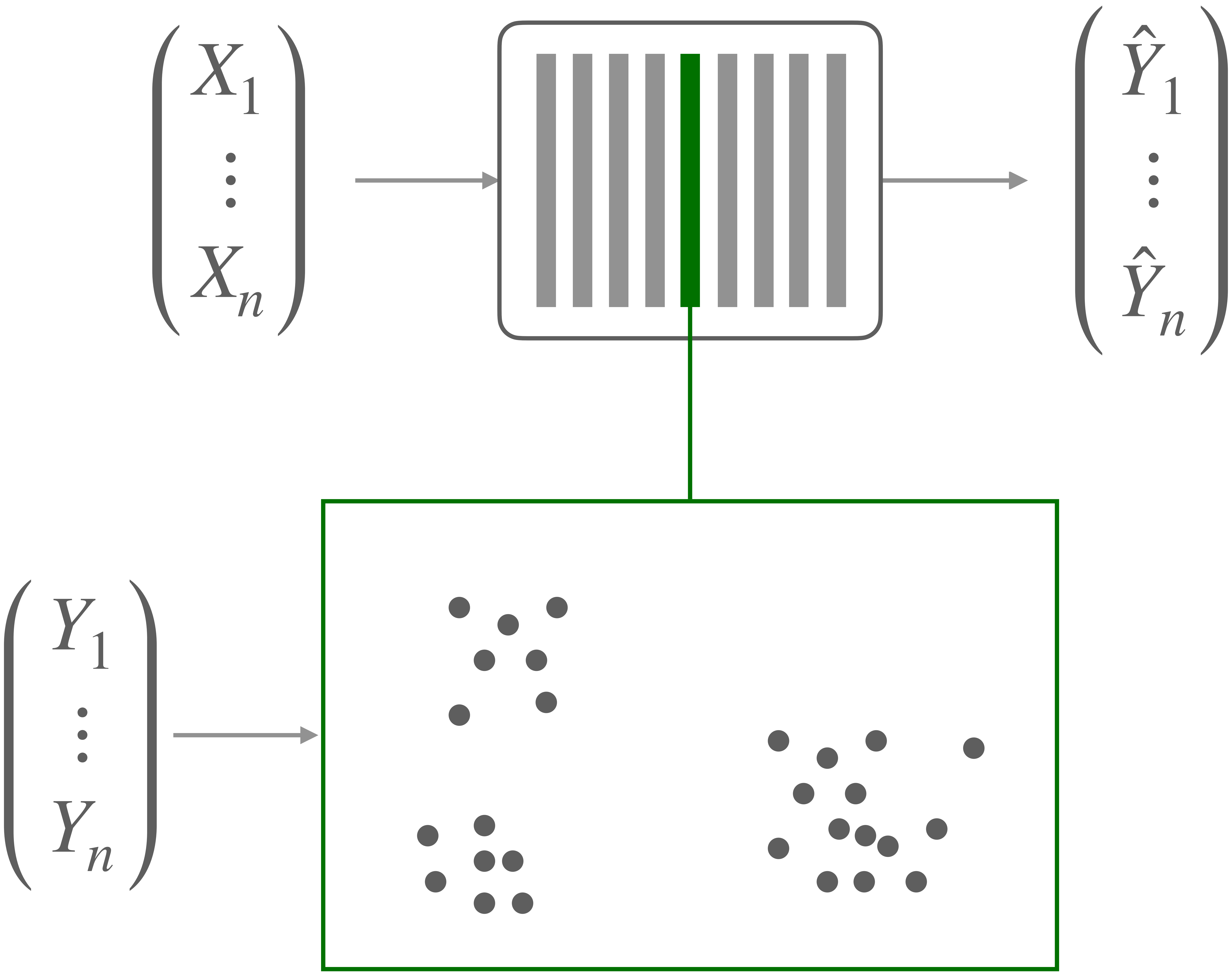

X_i
$\hat{Y}_i$


X_i
$\hat{Y}_i$

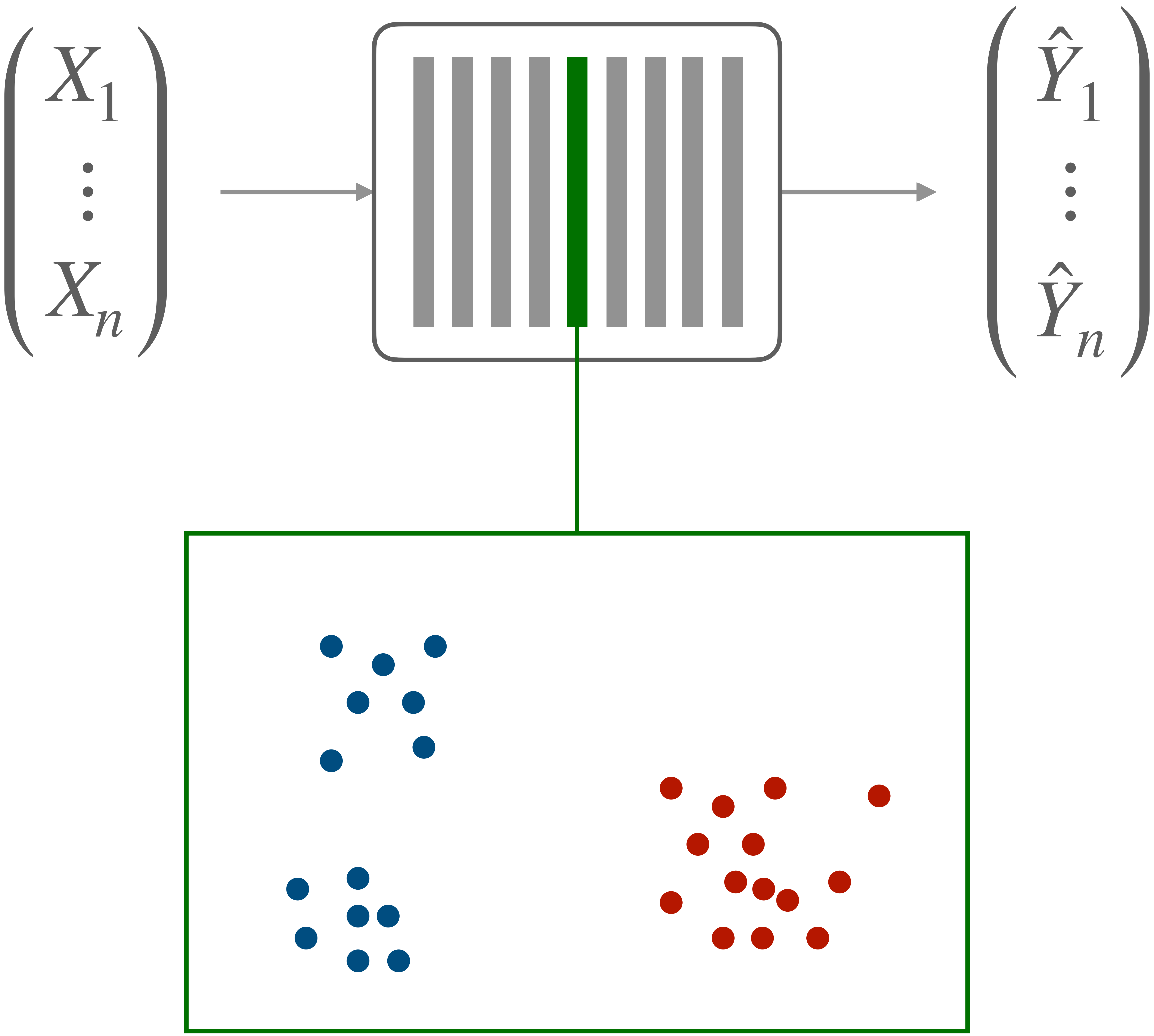

X_i
$\hat{Y}_i$

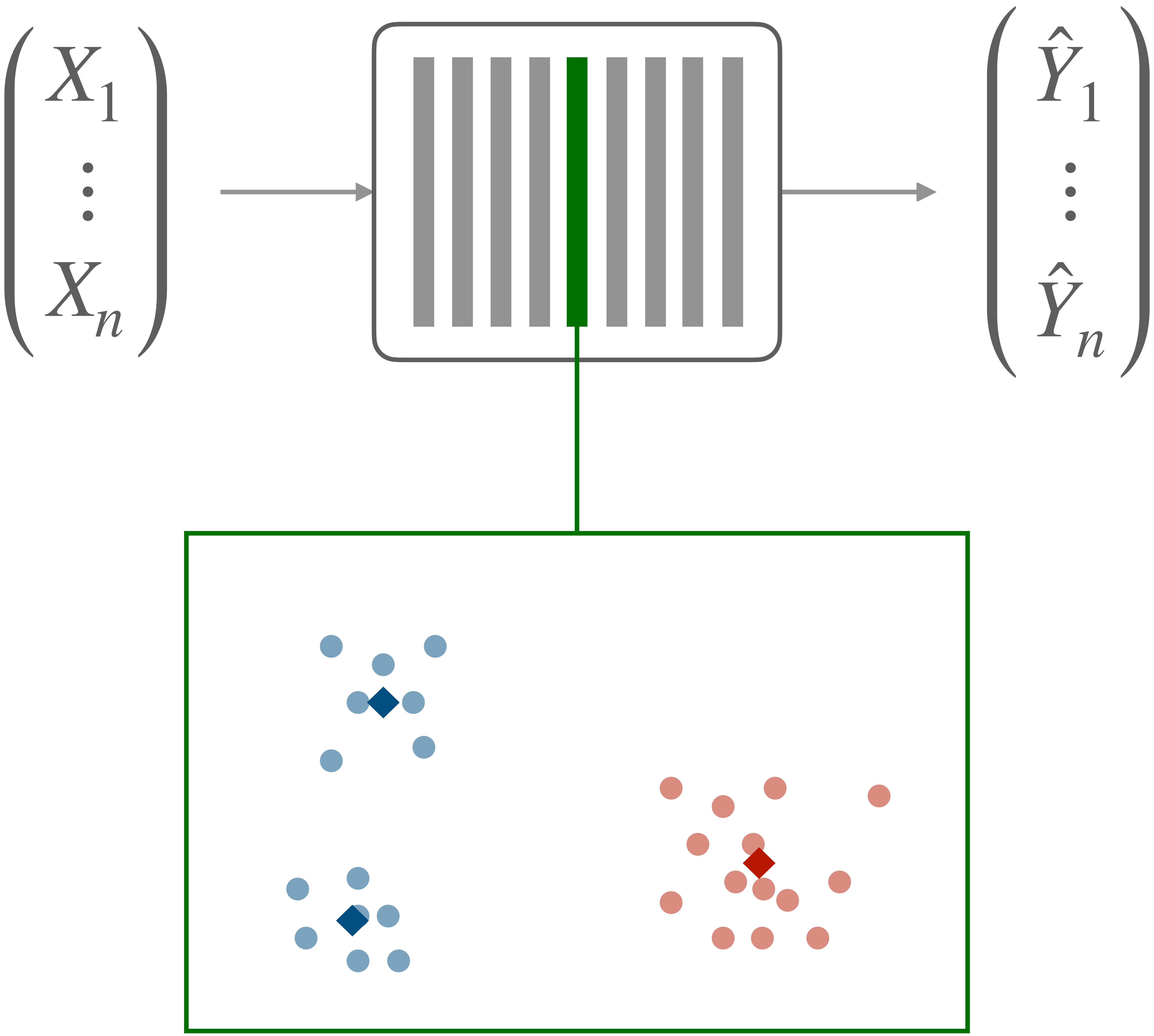

X_i
$\hat{Y}_i$

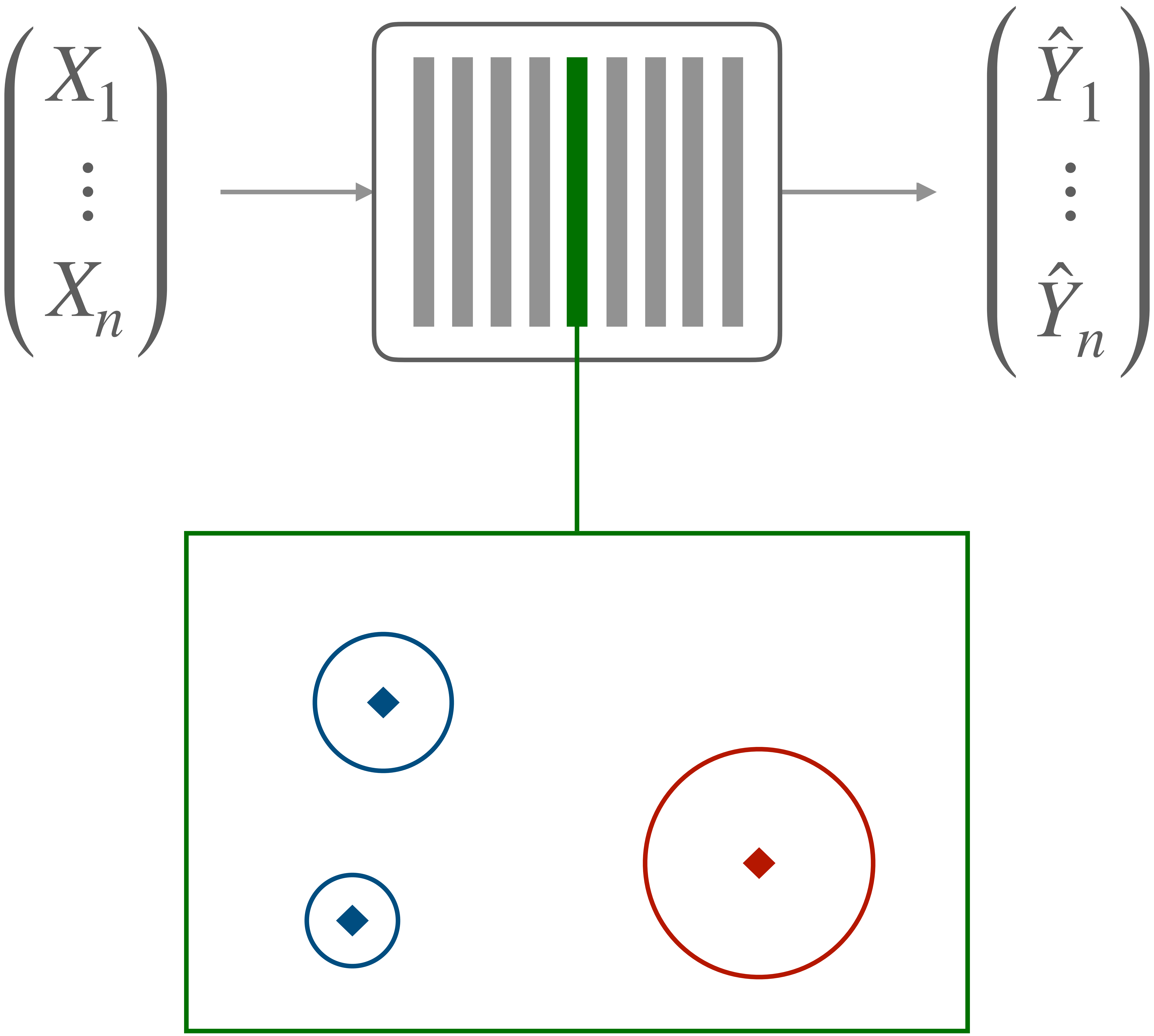

X_1
X_n
$\hat{Y}_1$
$\hat{Y}_n$

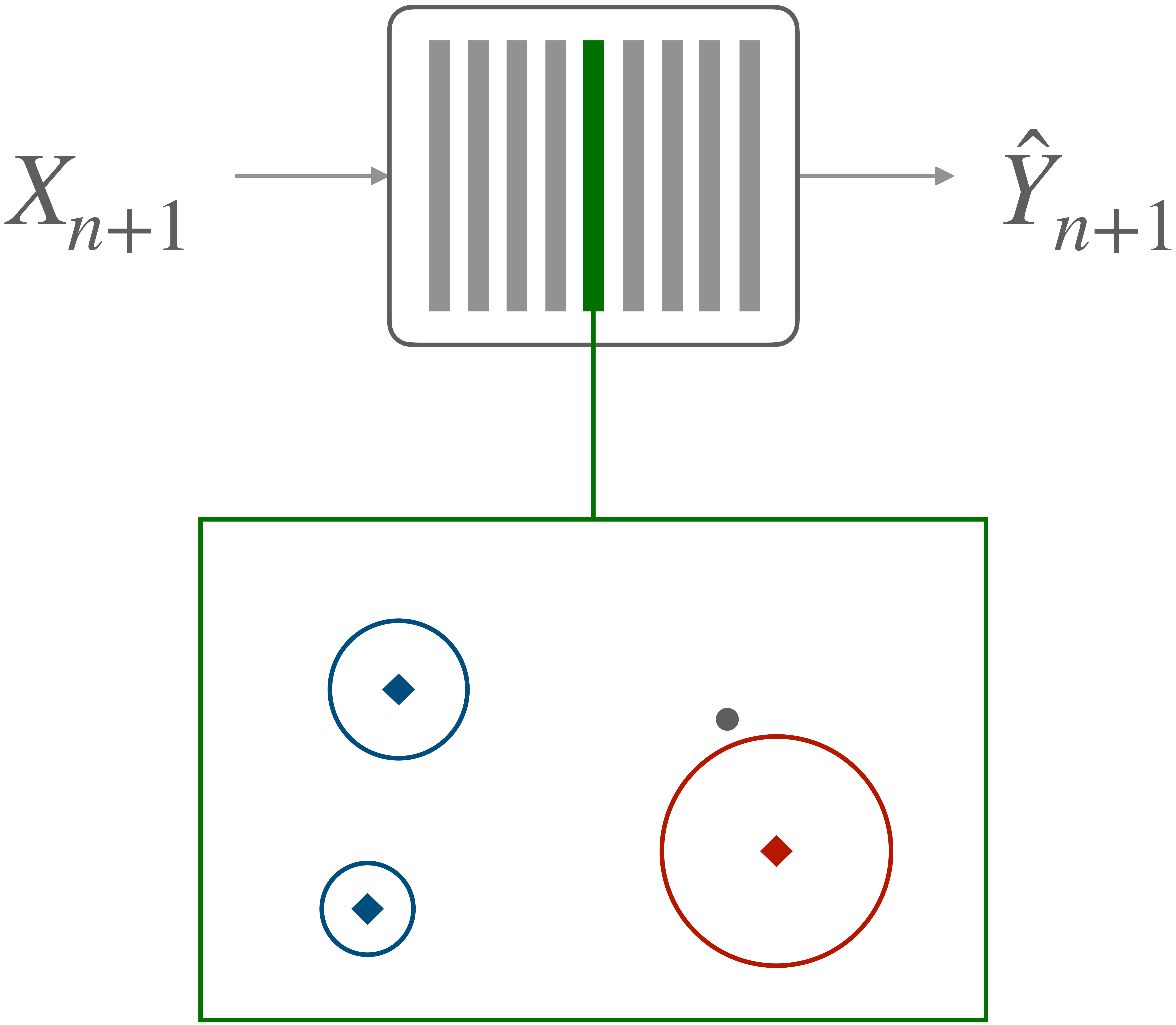


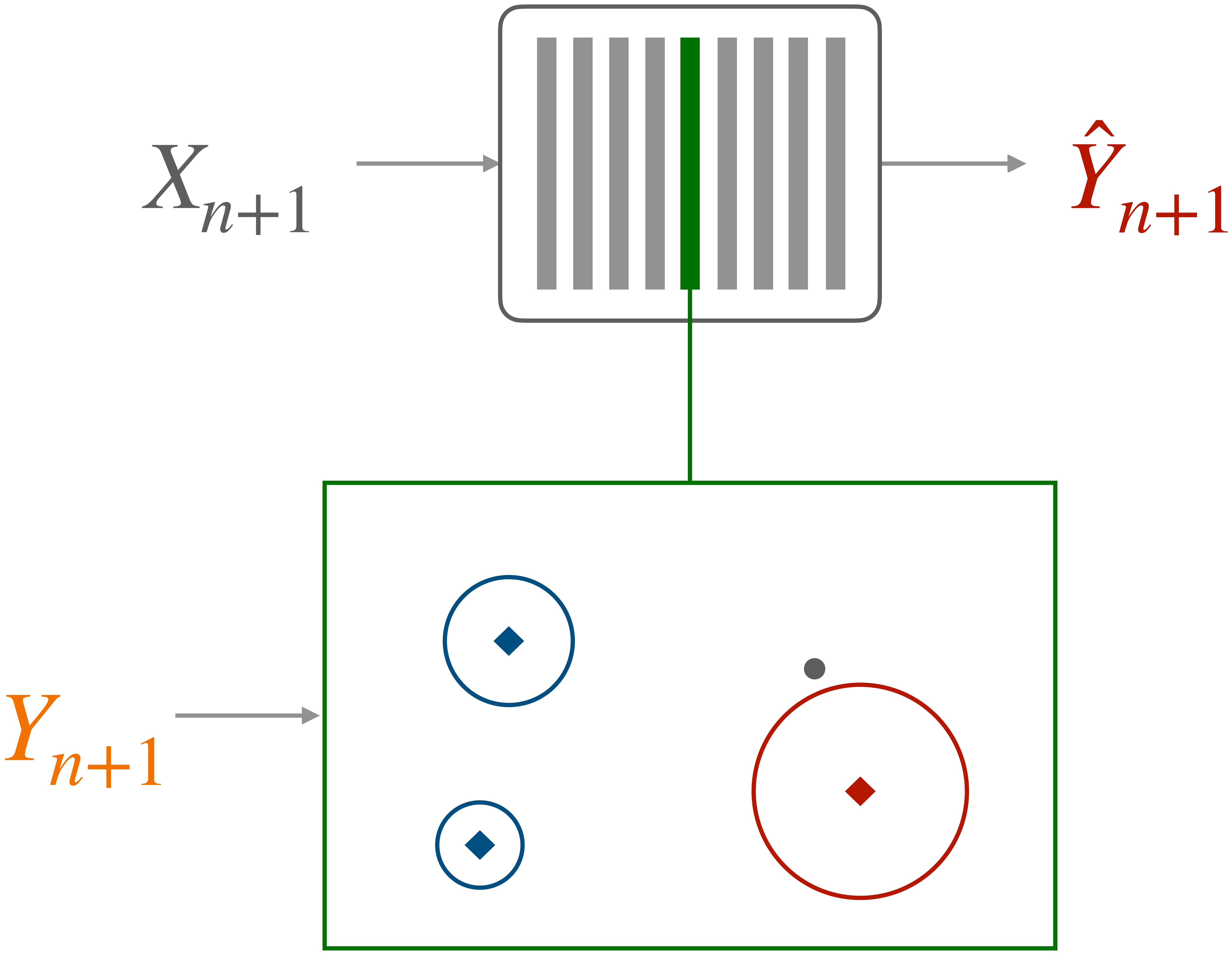
X_1
X_n
$\hat{Y}_1$
$\hat{Y}_n$
Y_1
Y_n

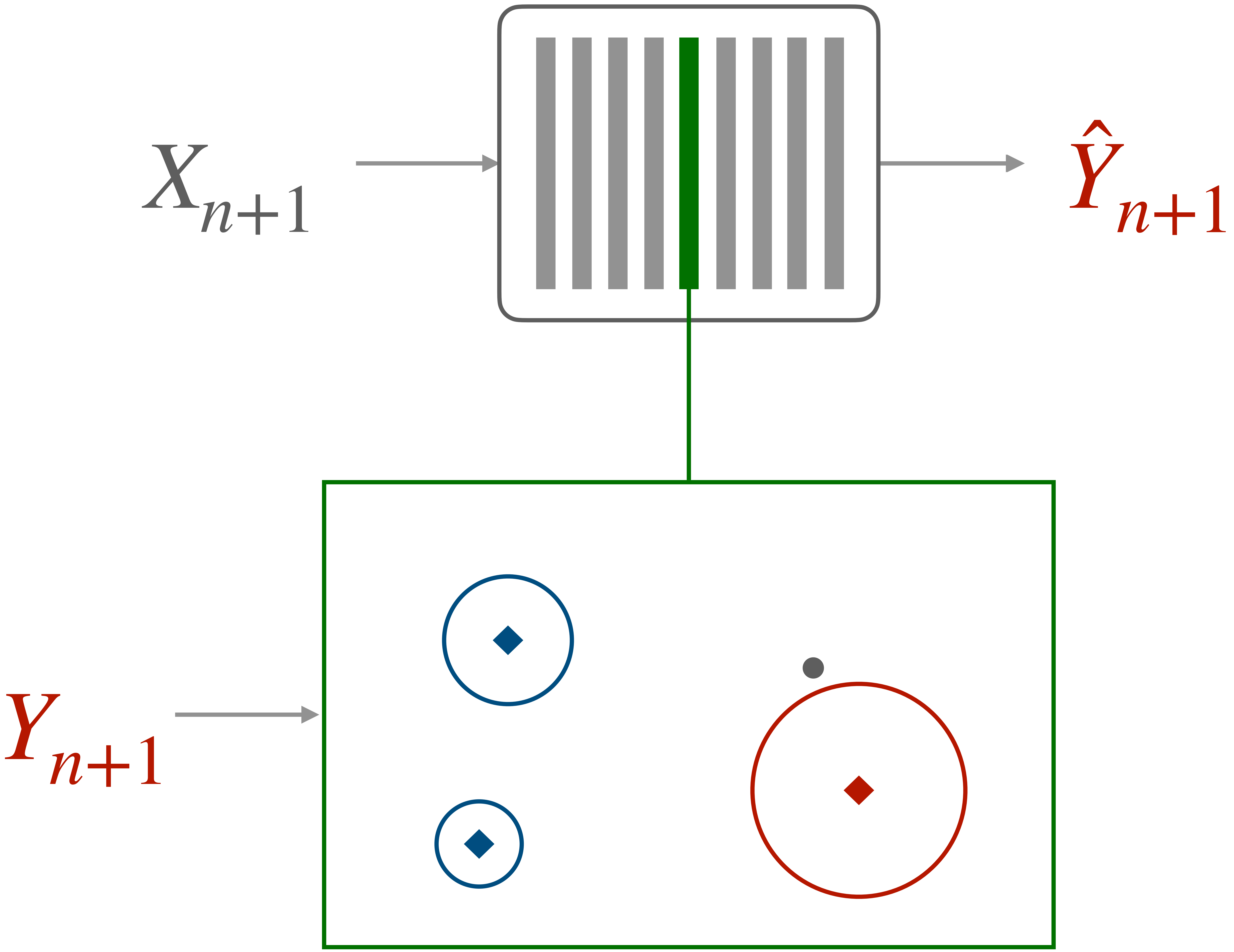

X_1
X_n
$\hat{Y}_1$
$\hat{Y}_n$

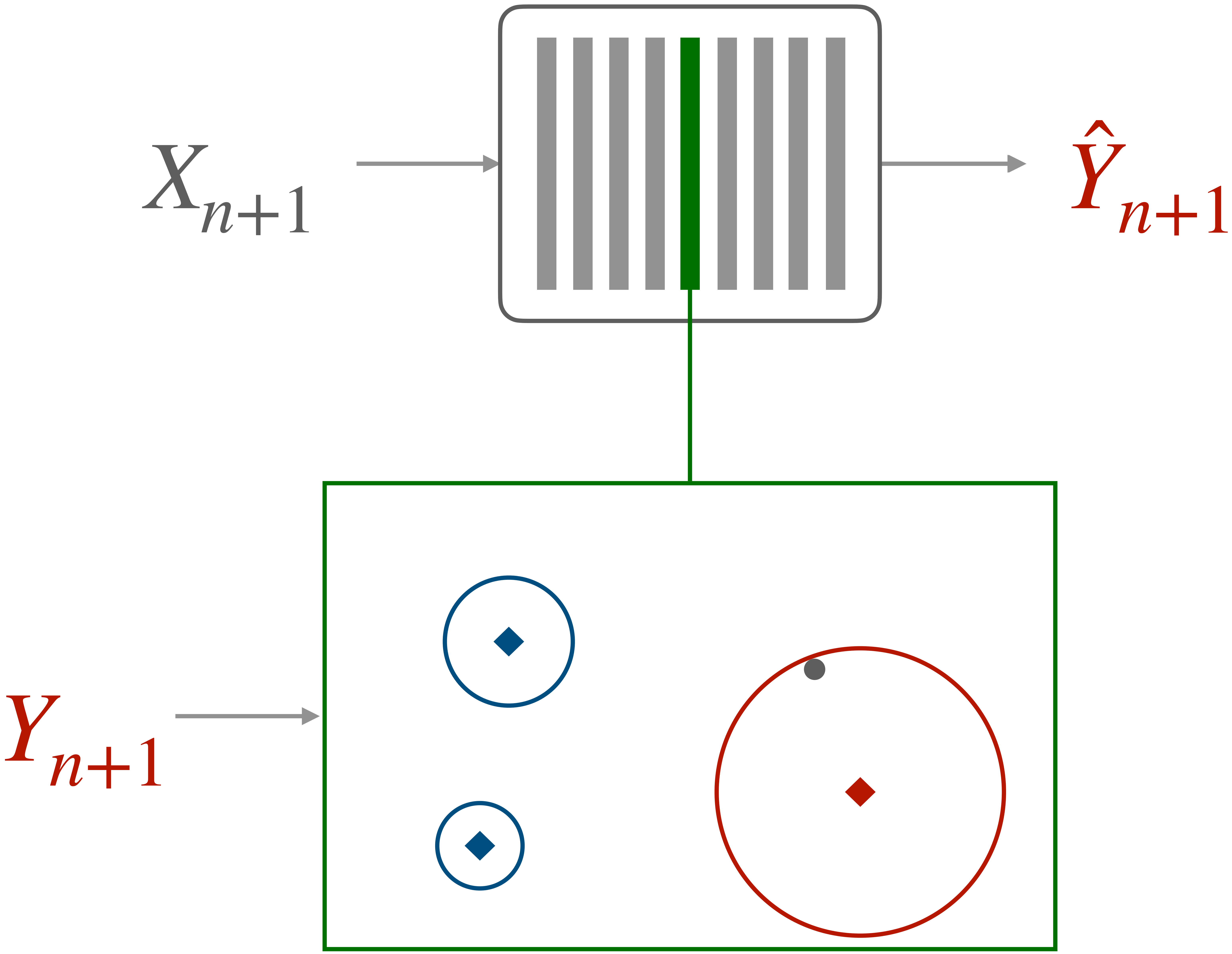

$\begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}$
$\begin{pmatrix} \hat{Y}_1 \\ \vdots \\ \hat{Y}_n \end{pmatrix}$


X_1
X_n
$\hat{Y}_1$
$\hat{Y}_n$


X_{n+1}
$\hat{Y}_{n+1}$

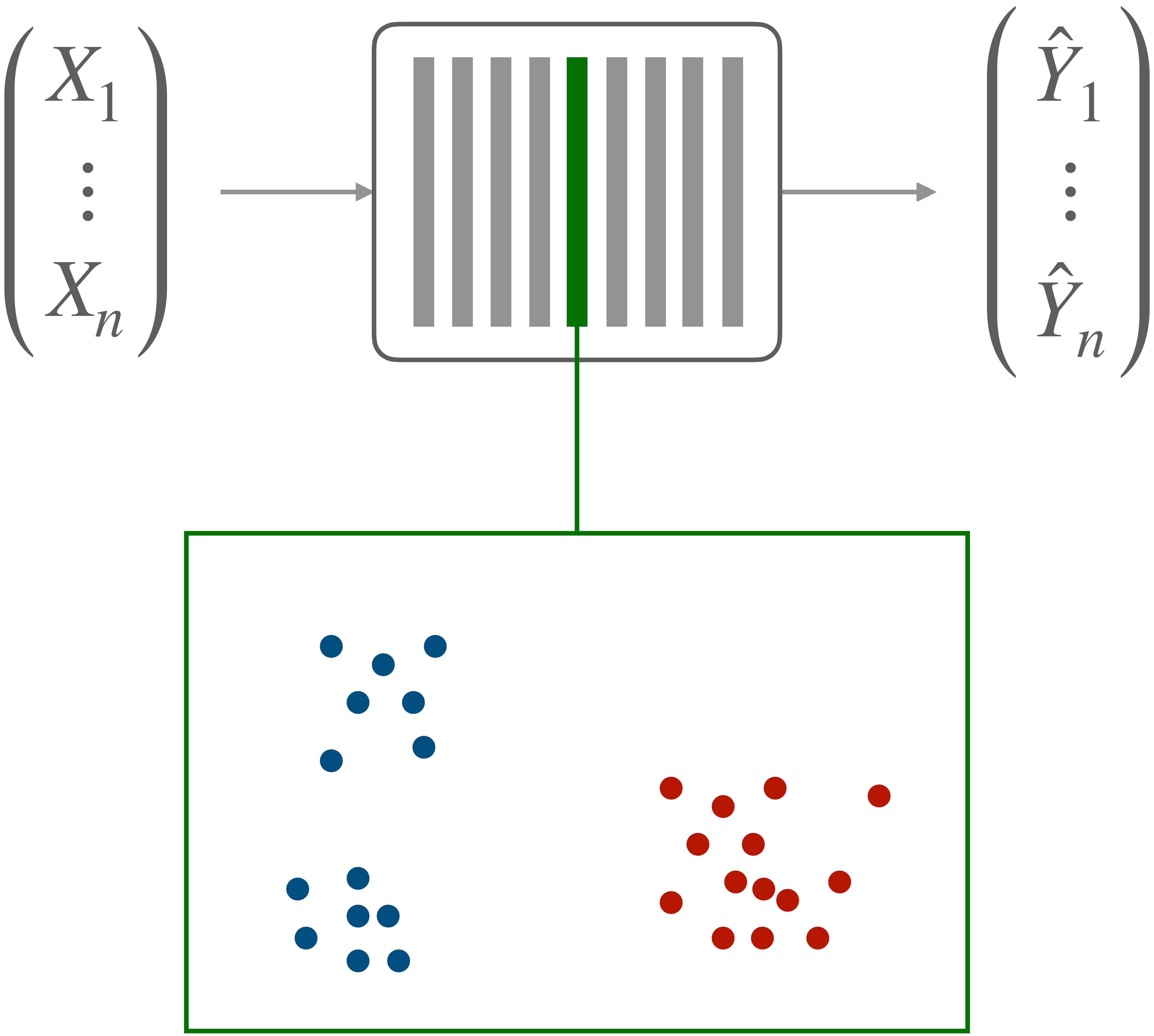

X_{n+1}
$\hat{Y}_{n+1}$
Y_{n+1}

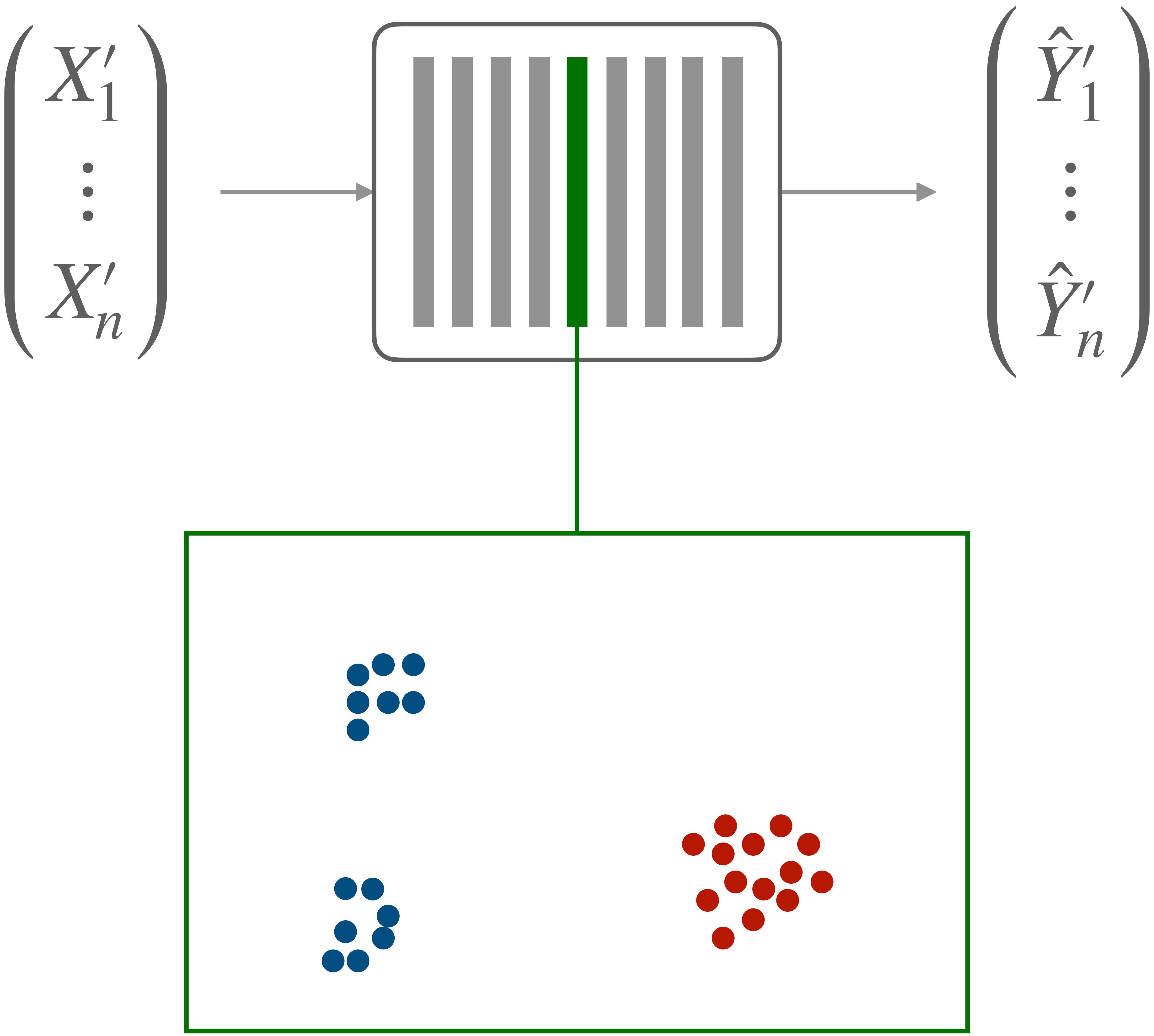

X_{n+1}
$\hat{Y}_{n+1}$
Y_{n+1}


X_{n+1}
$\hat{Y}_{n+1}$
Y_{n+1}

Monitor Friendly.

Encourage Closeness.

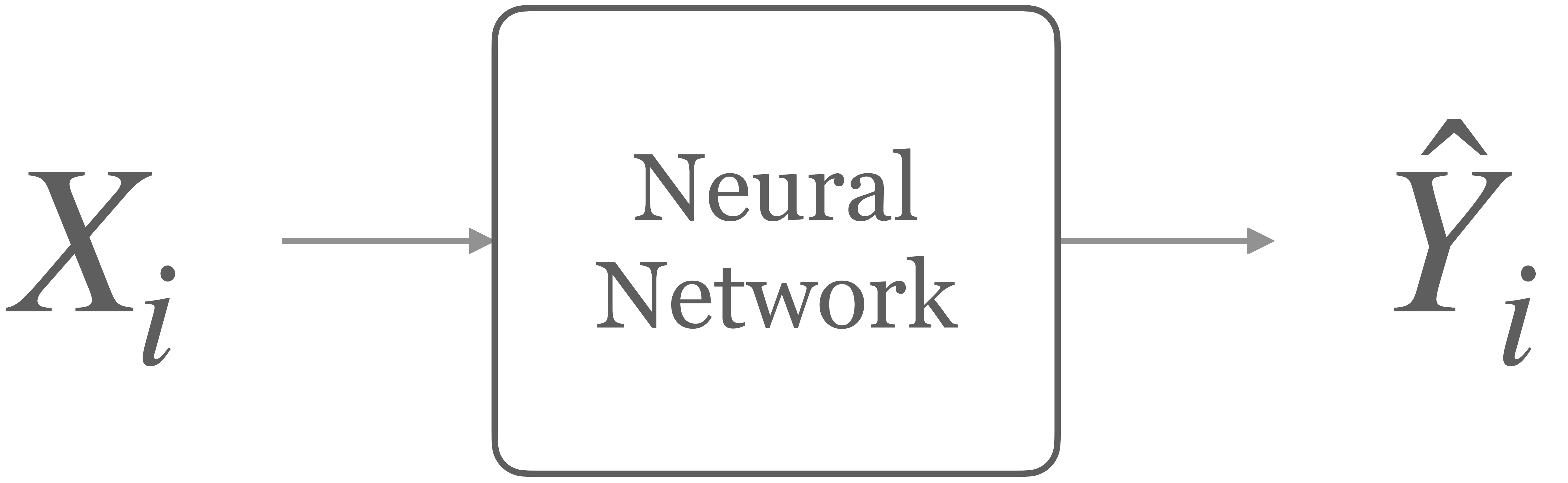

X_1
X_n
$\hat{Y}_1$
$\hat{Y}_n$

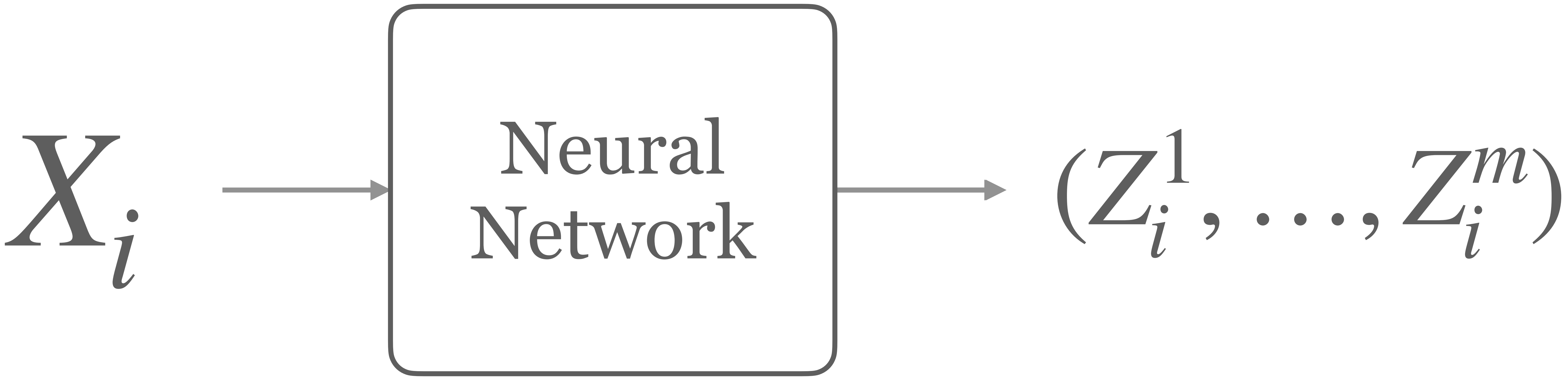

$\begin{pmatrix} X'_1 \\ \vdots \\ X'_n \end{pmatrix}$
$\begin{pmatrix} \hat{Y}'_1 \\ \vdots \\ \hat{Y}'_n \end{pmatrix}$

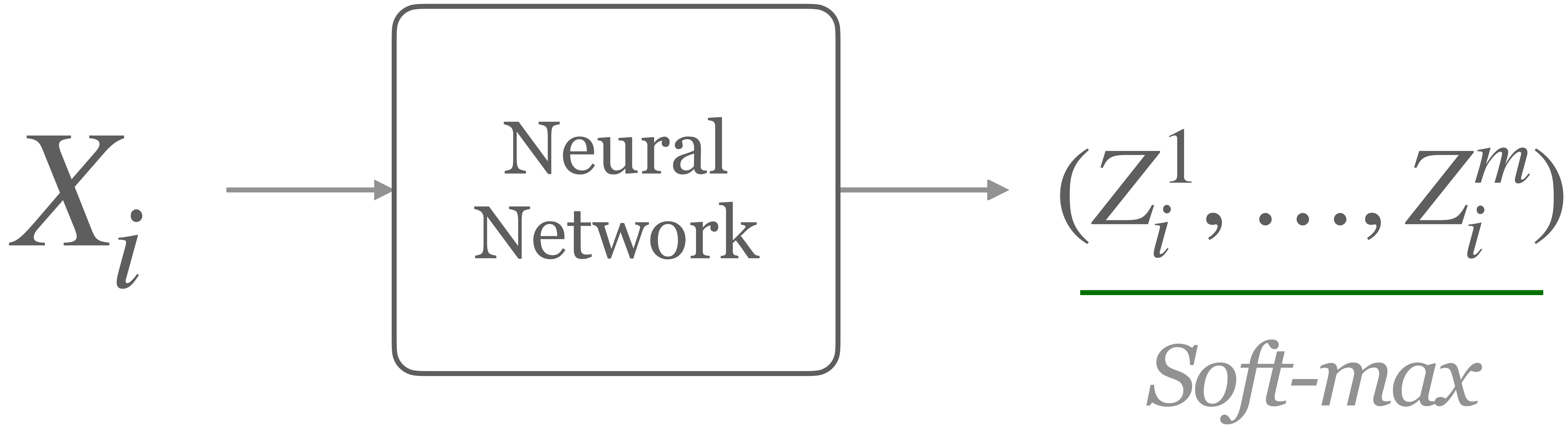
Out-of-specification Detection.

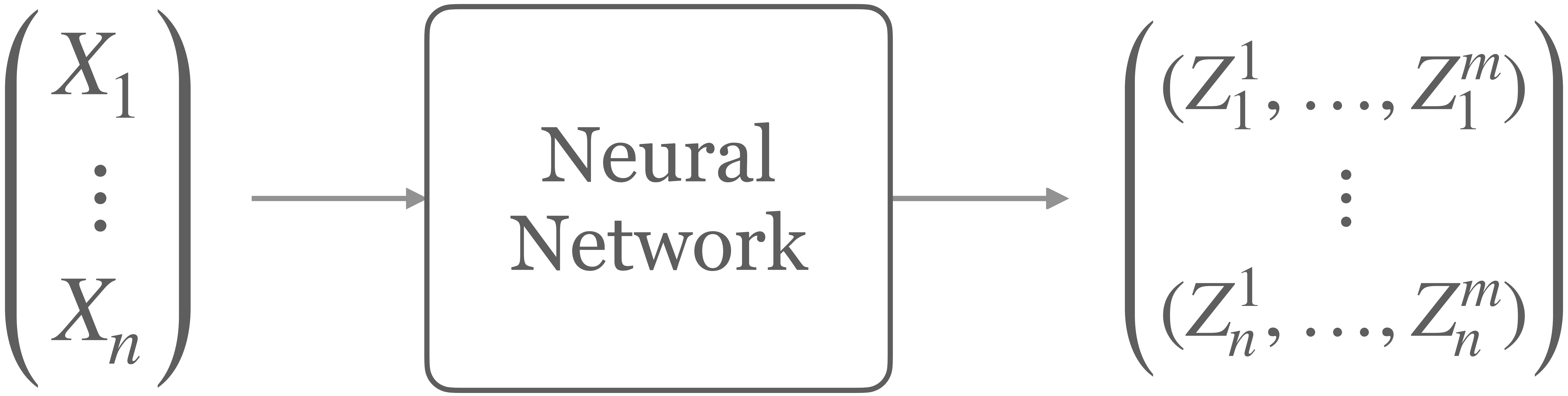
On Batches.

Rotation work based on: KS(conf). IJCV (2020)


X_i
Neural Network
$\hat{Y}_i$

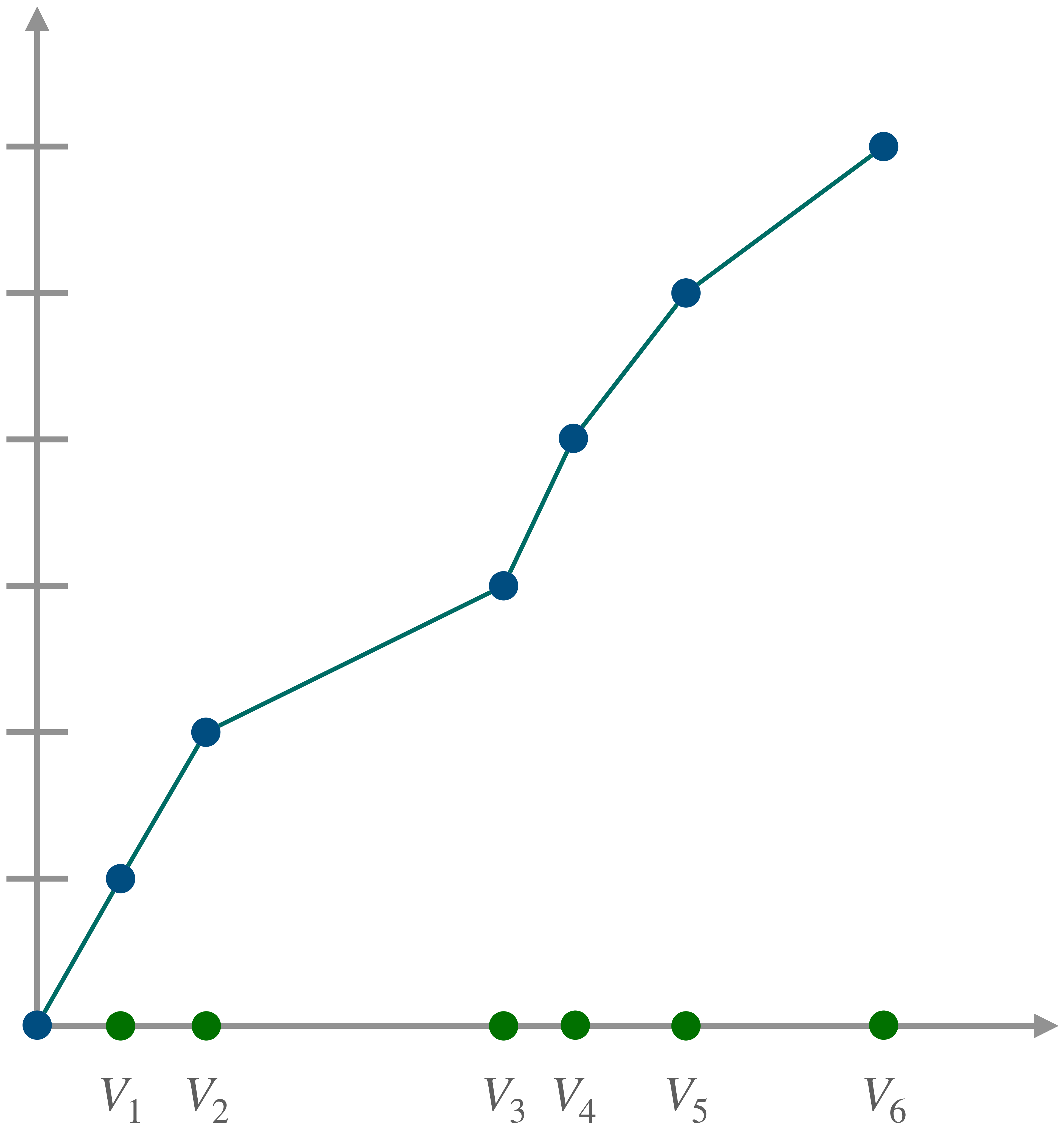

X_i
Neural
Network
$(Z_i^1, \ldots, Z_i^m)$


X_i
Neural Network
$(Z_i^1, \ldots, Z_i^m)$
Soft-max



Calibration

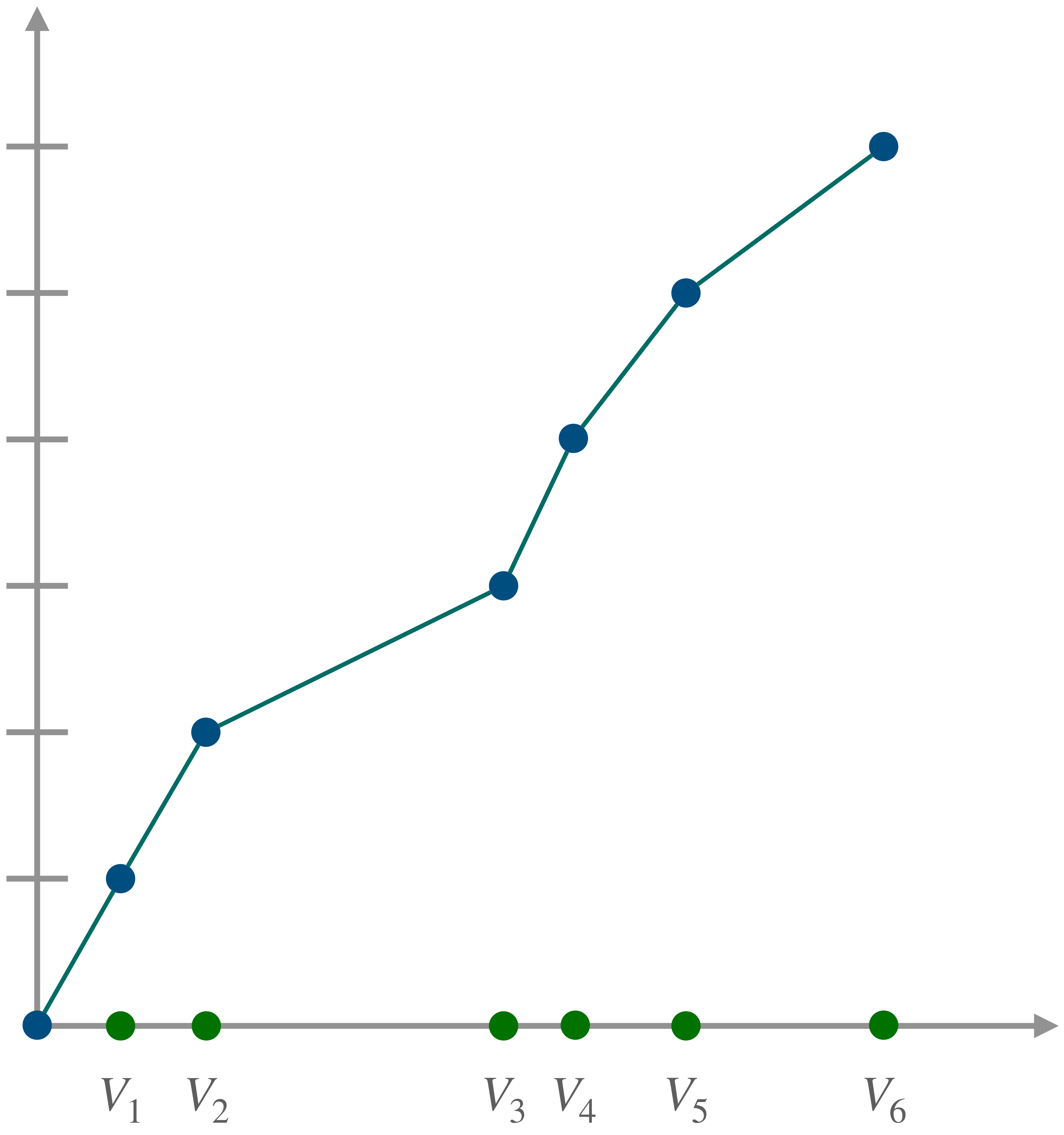
$$\begin{pmatrix} (Z_1^1, \ldots, Z_1^m) \\ \vdots \\ (Z_n^1, \ldots, Z_n^m) \end{pmatrix} \xrightarrow{\text{sort}(\tau(\,\cdot\,))} \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix}$$



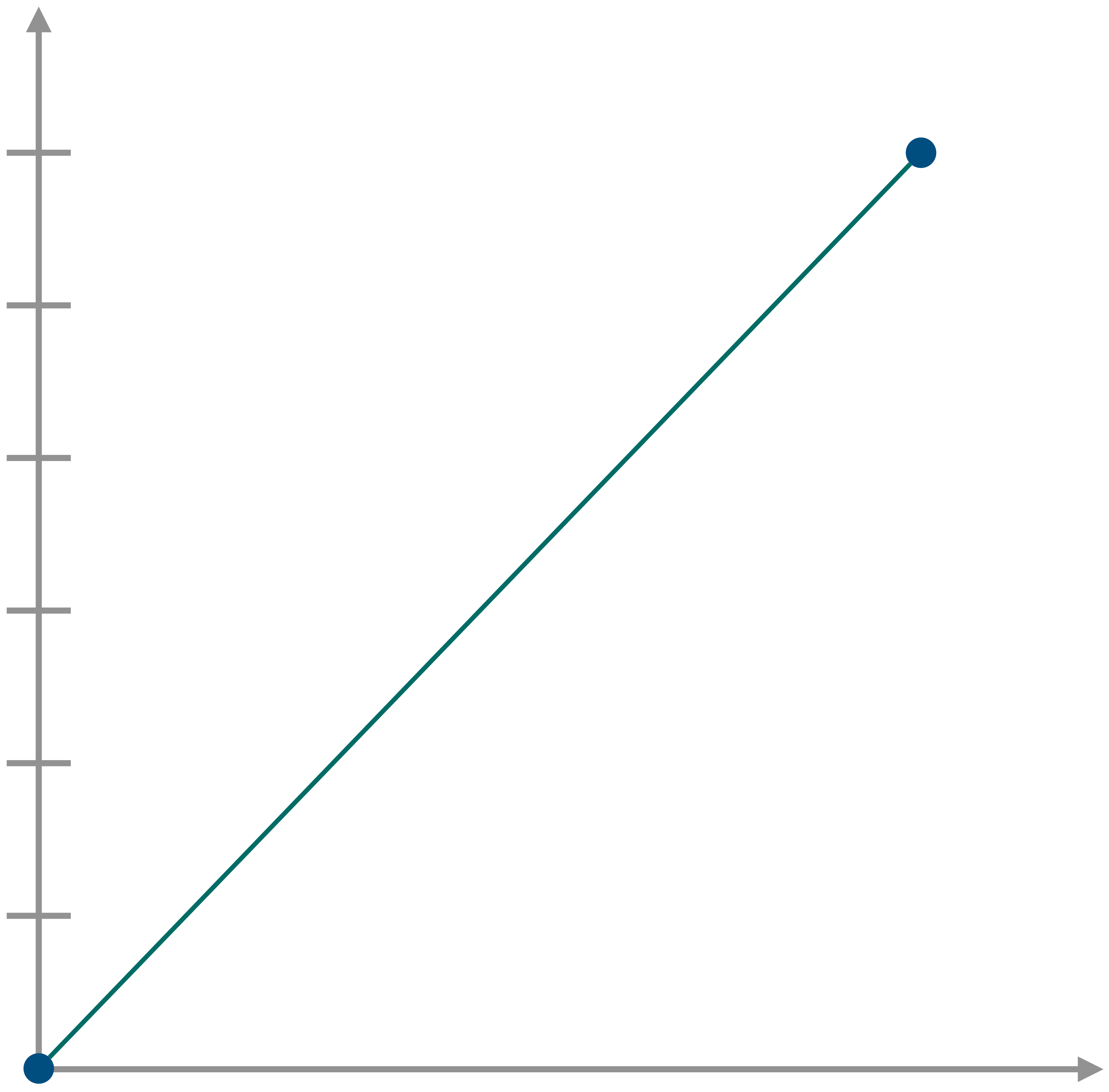
V_1
V_2
V_3
V_4
V_5
V_6

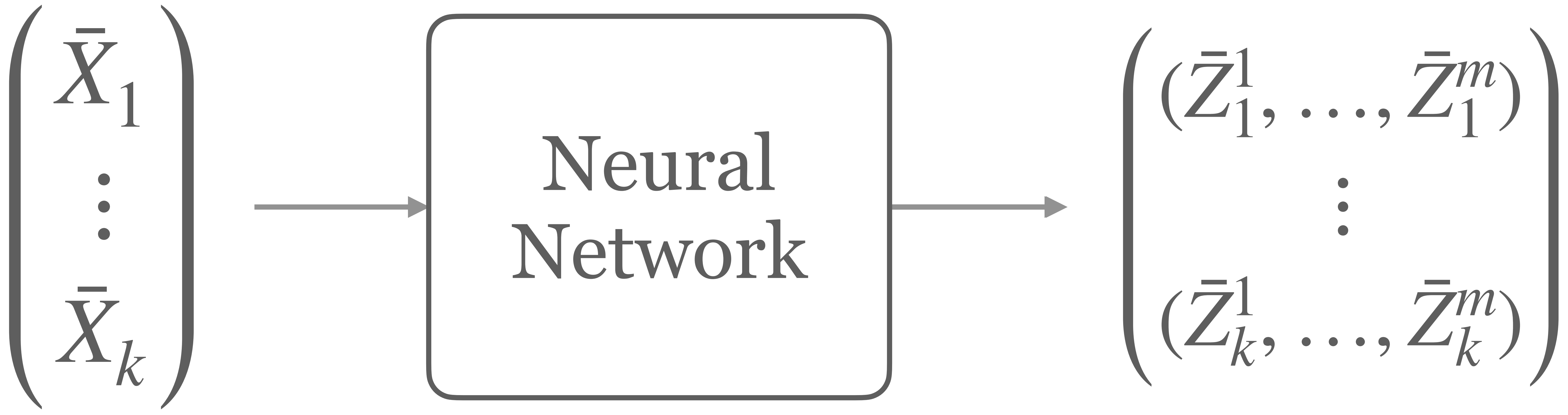
$$F :=$$

CDF



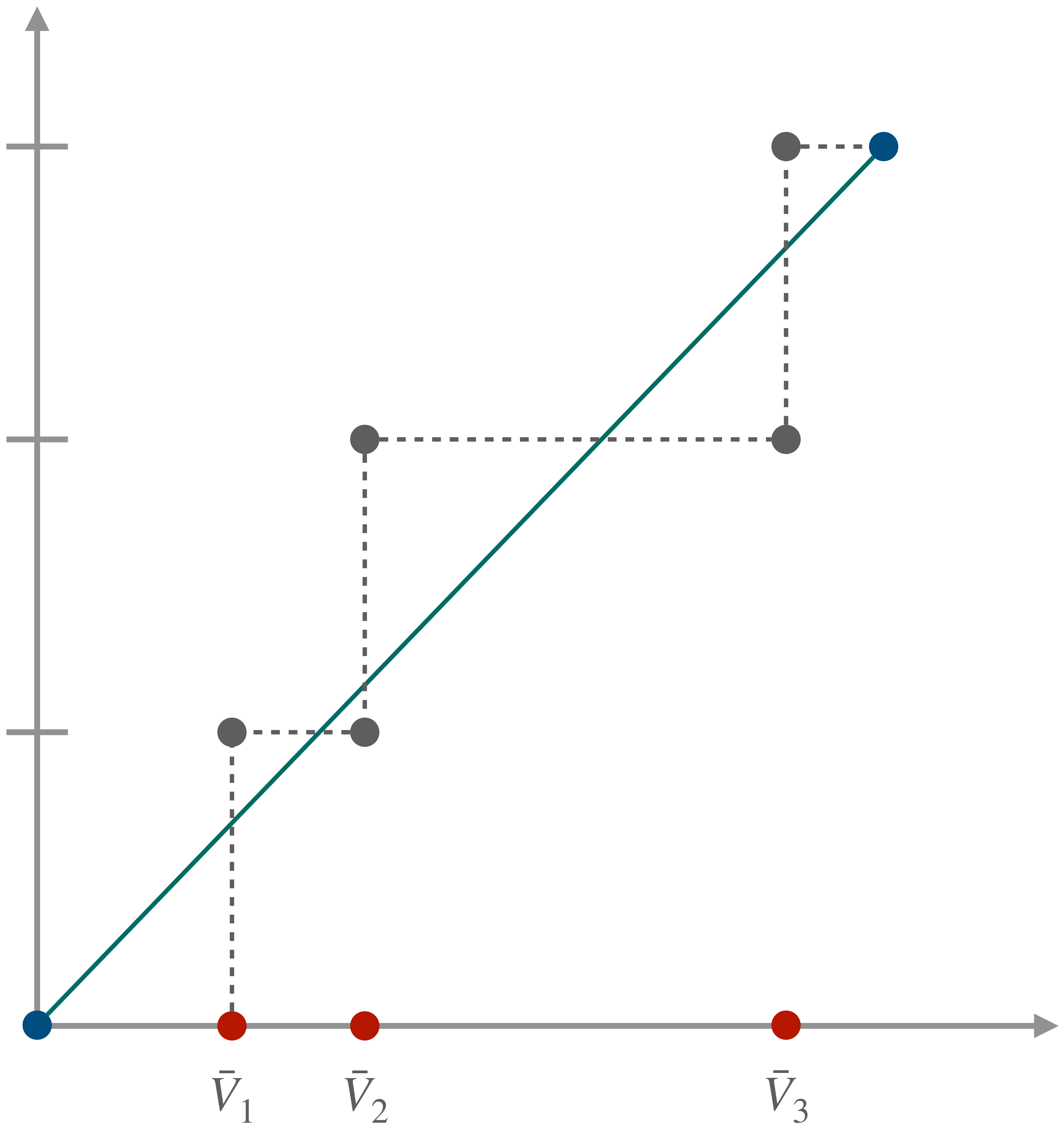
$$F\begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix}$$




$\begin{pmatrix} \bar{X}_1 \\ \vdots \\ \bar{X}_k \end{pmatrix}$
Neural Network
$\begin{pmatrix} (\bar{Z}_1^1, \ldots, \bar{Z}_1^m) \\ \vdots \\ (\bar{Z}_k^1, \ldots, \bar{Z}_k^m) \end{pmatrix}$

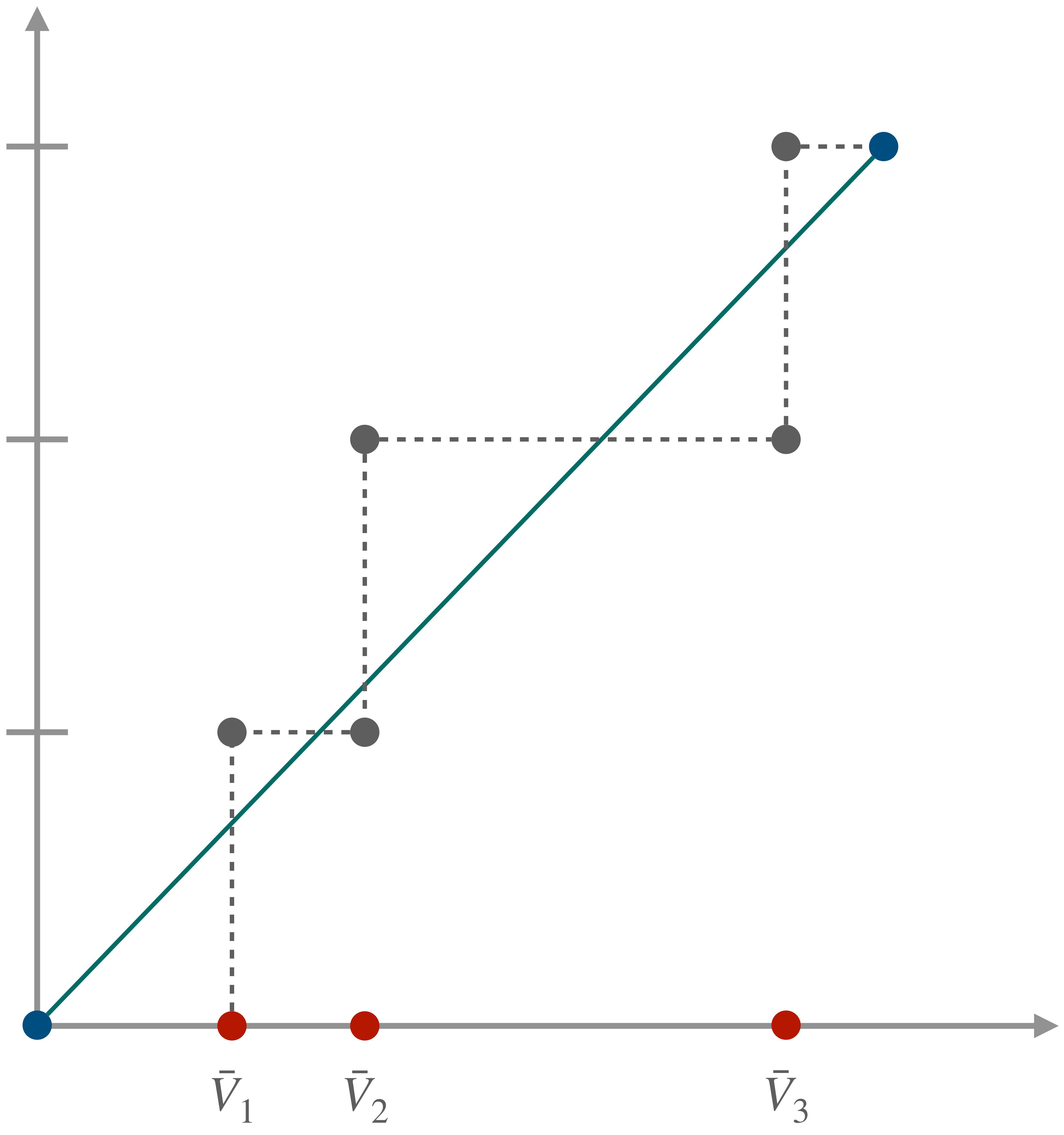
Runtime

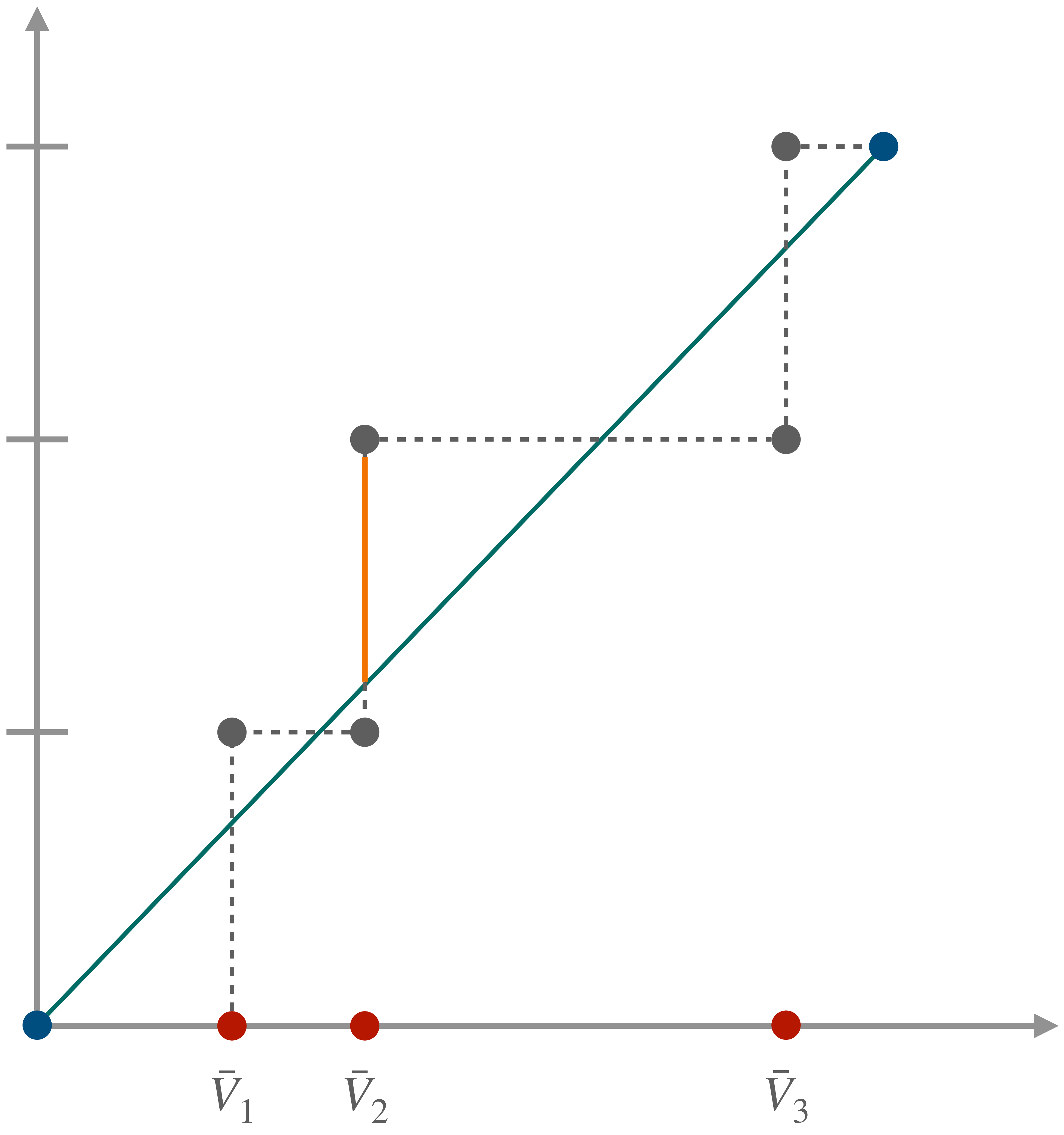
$$F\begin{pmatrix} \bar{V}_1 \\ \vdots \\ \bar{V}_k \end{pmatrix}$$


$\bar{V}_1$
$\bar{V}_2$
$\bar{V}_3$

$$KS(F(V), F(\bar{V})) \leq \theta_\alpha$$

Kolmogorov–Smirnov


$\bar{V}_1$
$\bar{V}_2$
$\bar{V}_3$



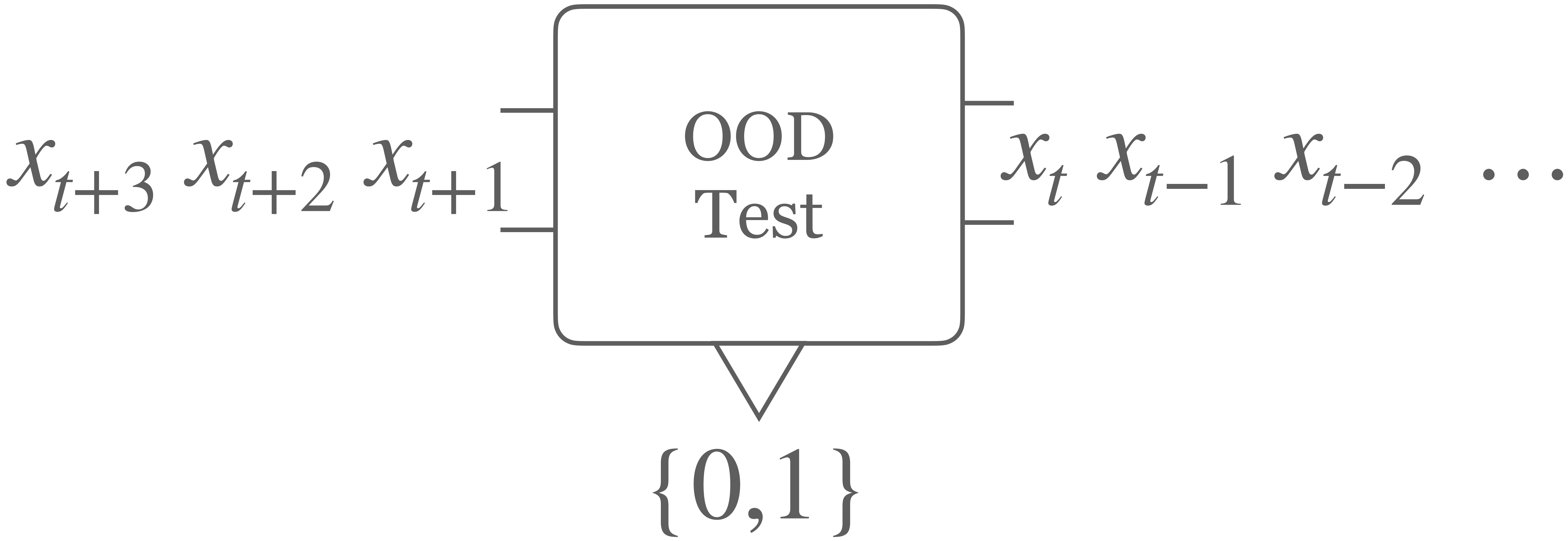
$\bar{V}_1$
$\bar{V}_2$
$\bar{V}_3$

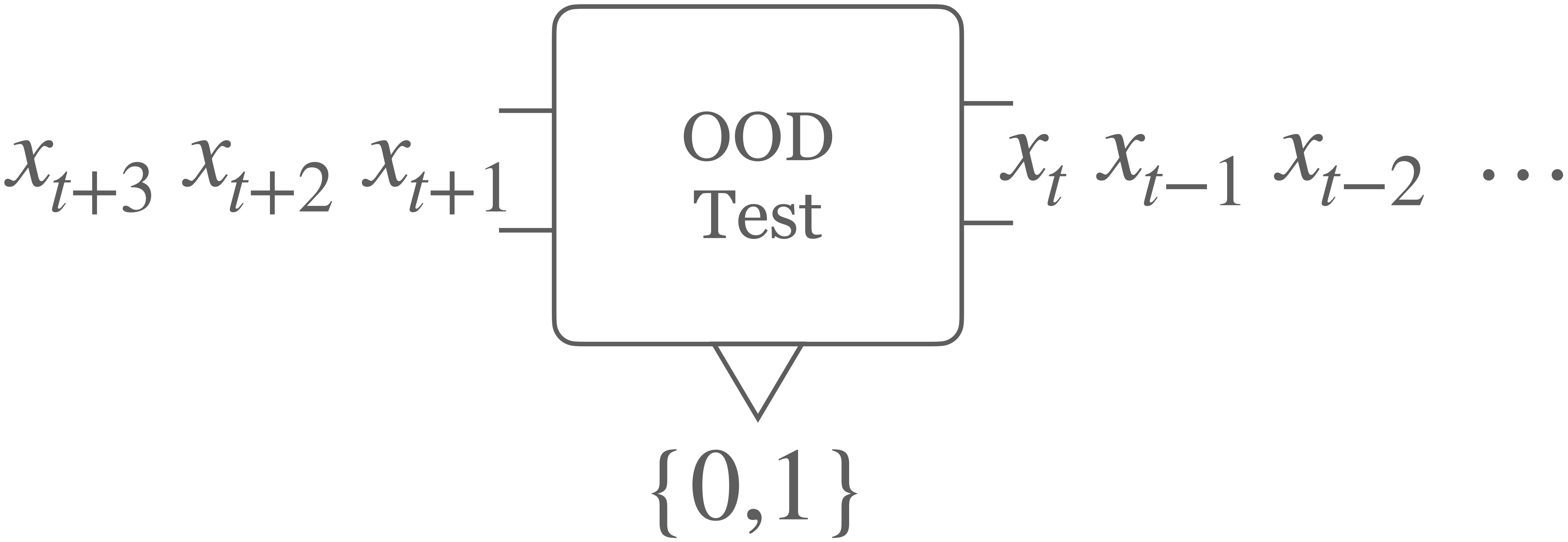
Tuneable Performance.

Due to control over KS error.

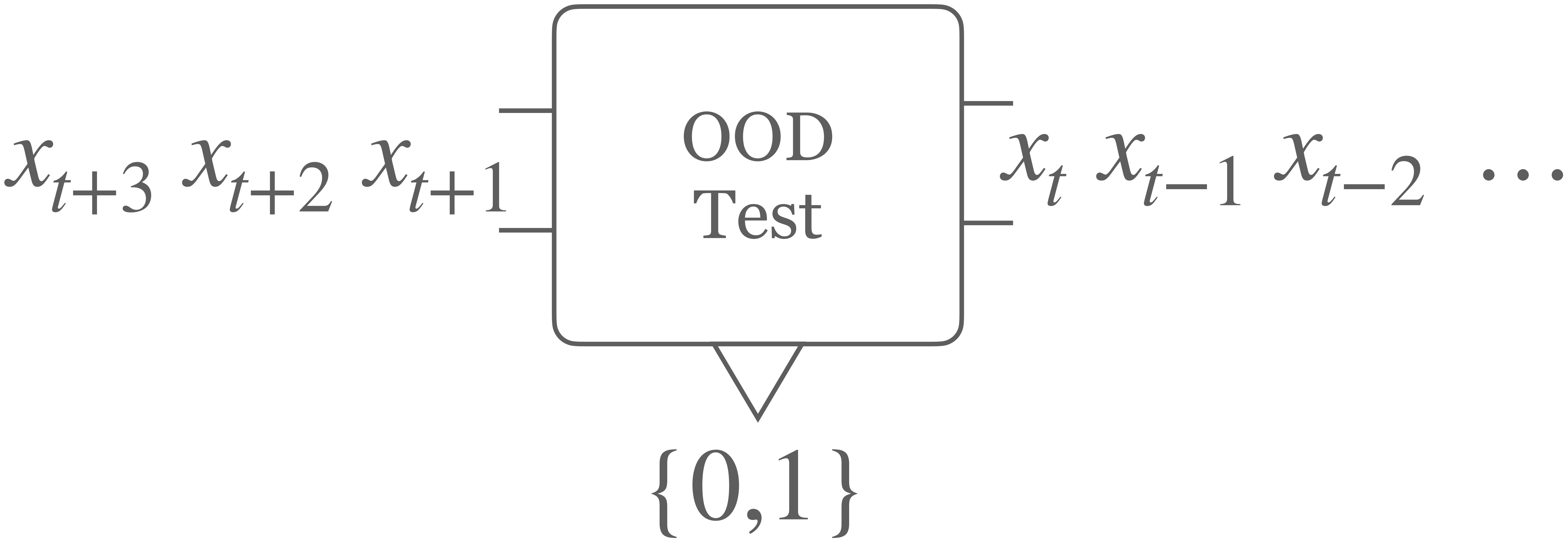
Possible Direction.

Investigate out-of-distribution detection (OOD) in a sequential setting.


x_{t+3} x_{t+2} x_{t+1}
OOD
Test
x_t x_{t-1} x_{t-2} $\cdots$
$\{0,1\}$



Sequential Hypothesis Testing



Sequential Hypothesis Testing

– inflate false positive rate

+ information

Summary.

Time to revise.

Quantify Uncertainty…

…for online monitoring;
…for planning;
…for verification.

Possible Directions.

At first glance.

Use statistical monitoring to quantify and reduce the uncertainty in the world model.

Improve safety in planning by quantifying state and/or model uncertainty.

Investigate out-of-distribution detection in a sequential setting.